Key group analysis based on DDA method for rock slope stability analysis

Mir Hosein Shahami[1], Alireza Yarahmadi Bafghi[*1], Mohammad Fatehi Marji[1]

1- Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran

* Corresponding Author: *ayarahmadi@yazd.ac.ir*
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Abstract

Inhomogeneity and discontinuities play a key role in the resistance and behavior of rock masses. Today engineers have a wide range of methods to analyze the stability of rock slopes. Due to its simplicity and speed of evaluation, static analysis methods continue to play a special role in the stability assessment of jointed rock slopes. One of the most well-known static methods used in the stability analysis of rock slopes is the Key Block method (KBM), which is based on key block finding and analysis. In this method, if none of the key blocks are unstable, it implies that rock mass is stable. Occasionally, the combination of several stable blocks has led to the formation of a group of blocks that sometimes leads to instability. Therefore, the stability analysis of the jointed rock masses leads to study groups of blocks that are potentially dangerous for the stability of a rock slope. The Key Group method (KGM), with its progressive approach, finds these critical groups and focuses the stability calculations on these groups. Until now, methods SKGM, PKGM, OKGM have been proposed to remove the limitations of this method and its development. In order to increase the efficiency, accuracy, and speed of this method and to develop it in three dimensions, it is decided to combine it with one of the numerical methods. The standard Discontinuous Deformation Analysis method (DDA) is an implicit method based on the finite element method. This is a sophisticated numerical method for modeling the quasi-static and dynamic behavior of rock block systems in discontinuous rock masses. The goal of this paper is to use the potency of the numerical method of DDA to analyze the candidate key group. For this purpose, the DDA computer program was developed with Mathematica programming language and combined with the KGM software. The resulting package, after selecting the key group by the KGM method, proceeds to analyze it with the DDA method. Two examples are solved illustrating the reasonable results and the efficiency of this developed method compared to that of the original KGM and SKGM. The results validated the proper accuracy and good performance of the procedure developed in this research.

1- NTRODUCTION

Discontinuities are a considerable source of uncertainty and variability in the engineering design of rock slopes. Rock slopes are often found in complex and rugged terrains such as mountainous areas, where strength and geometric properties of the discontinuities are subjected to change within a rock slope [1, 2, 3, 4, 5, 6, 7, 8]. Various static (limit equilibrium), quasi-static and dynamic methods are used to analyze the stability of slopes in discontinuous rocks. Most of these methods have been developed to address the influence of the inherent variability of strength properties for slope stability assessment in a realistic manner [8, 9, 10, 11, 12]. One of the most well-known limit equilibrium methods for the analysis of the slope stability of jointed rocks is the Key Block Analysis (Key Block Theory) method, which uses two graphical methods based on stereographic [13] and vector mapping [14] to investigate instability. This theory studies the displacement of rock blocks and analyzes the equilibrium of these blocks at various excavations. So far, many researchers have used this method to analyze the stability of unstable blocks created by a discrete fracture network [15, 16]. Based on this theory, moving one block creates space that moves the other constrained blocks. This can lead to a progressive failure, which sometimes happens rapidly. The key point in the key block method is that with the assumption that the key block is controlled, combining the key block with its neighbors can create a critical group that is talented to collapse. On the other hand, since this method only deals with key block analysis, if none of the blocks are identified as the key block, it results in the stability of the block system, whereas a group of blocks can be assumed to create instability when merged. One of the presented methods for solving the problems expressed about the key block method is the Key Group Method (KGM) of Yarahmadi and Verdel (2003) [17]. In an iterative and progressive analysis, this method examines the slope stability of the jointed rocks and finally identifies the critical key group that causes instability. The principle concepts used in the key block method are also repeated in this method and applied to identify the key group.

After introducing the key group method, its developments were presented as a probabilistic key group (PKGM) to investigate the uncertainty of the key group instability. After this, key groups based on Sarma's method (SKGM) were introduced for analyzing the movement of the blocks relative to each other by Yarahmadi et al. [18]. KGM was introduced by Emami et al. as an oriented key group to find the critical surface of failure [19] and then expanded by Norouzi et al. in three dimensions [20].

In the original key group method, the blocks of the key group after identification, are combined and form a single block. This means that the interaction between blocks in a key group is disregarded, and can cause many problems in analysis. To solve this problem, the Sarma-based Key Group method (SKGM) [18] was proposed by the authors. This method was proposed to overcome the problem of key group rigidity. In this method, the concept and equations of Sarma's method were used to analyze the interaction of blocks in a key group; but not with the real geometry of joints. The SKGM method has two fundamental problems; first, the slices used in this method do not match the actual joints of the rock mass necessarily. Second, as the problem becomes more complex, this method will not be able to perform complex analysis, for example, in terms of the presence of water in joints, stresses, curved joints, etc. Combining the key group method with numerical methods causes a direct focus on a group of blocks rather than all of the blocks of the rock mass that will increase the speed, accuracy, and capability of rock mass analysis. It also allows for a more accurate simulation of the rock slopes by applying in situ stresses, inertial forces, etc. The DDA numerical method was first introduced to model the dynamic behavior of the discontinuous and blocky systems by Shi [21, 22, 23, 24]. It uses block displacements as the main variable. This method, which is based on energy calculations similar to the finite element method, can overcome DEM method constraints [25]. The original method was two-dimensional and then the 3D-DDA method was presented [26]. Thereafter the three-dimensional model of this method was considerably researched by the researchers [27, 28, 29].

This paper, it is attempted to solve the problem of rigidity in the key group by using the concept and formulation of the DDA method. Also, in the developed method, unlike the SKGM method, the actual joint geometry is used to model the blocks of a key group. In addition, the use of the DDA numerical method in key group analysis minimizes the limitations of the analysis. To evaluate the results of the developed method, the examples used in the validation of the KGM and SKGM methods are employed [18]. The results validated the proper accuracy and good performance of the procedure developed in this research.

2- The Key Group Method (KGM)

There are various types of failures in the rock slopes. A closer look at all of these failures shows that all these failures result from several major types. The major failures are: block sliding along one surface; block sliding on two surfaces; rotation of the block around one of its vertexes; and rock failure due to shear or bending stresses. The failures in the rock slopes are a combination of the major failures and may be associated with creating new cracks. For example, the failure of the rock slope in Fig. (1) is due to the simultaneous displacement of four different blocks, each of which is carried out by specific mechanisms. In this case, it is very likely to stop all displacements by preventing the movement of block 1. But at the same time, the combination of block 1 with its neighboring blocks may cause instability on a larger scale. Therefore, it cannot be denied the impossibility of predicting some of the failures by the simple key block method. The KGM method was introduced to overcome the mentioned weaknesses of the KBM method. The key group method is based on progressive stability analysis. So, if the blocks are considered together, they can form a key group that potentially is more dangerous than a single key block. The following conditions should be considered to determine key groups.

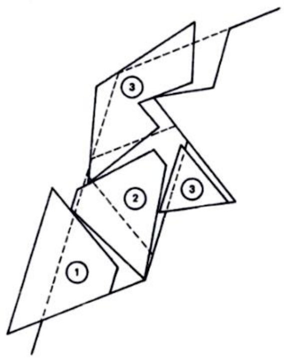
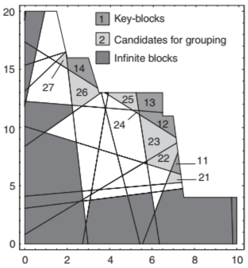


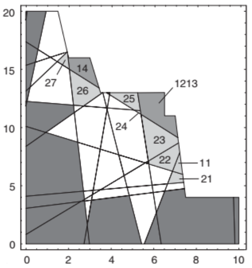
Fig. 1-. Progressive failure of a rock slope

1- A key group contains at least one primary key block (A key block is a block that has a free surface or extractive surface and is the key to moving other blocks). Therefore, when searching for key groups, this result is the primary condition.
2- A key group must have a key property (In other words, should be active, finite, and geometrically movable).

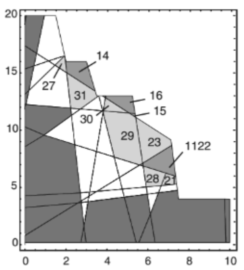
For example, Figure (2) schematically illustrates four steps of the grouping technique performed on an assumptive jointed rock slope. At the first level (Fig. 2a), four key blocks are identified. If the key block method is used for analysis, none of the blocks will be unstable and the analysis will end at this stage. But using KGM method five candidate key groups (22 + 11, 12 + 12, 24 + 12, 25 + 13, 14 + 27) can be identified. At this level, only the key group including key blocks 12 and 13 will be unstable, so this group will be deleted in the next step. Blocks 11 and 22 are then combined to form the key group 1122 that has the least factor of safety (Fig. 2c). At this level, there are three grouping positions with group 1122 (1122 + 23, 1122 + 28, 1122 + 21) that only the group 1122 + 23 has the characteristics of a key group. At the same time in this level, three other key groups can be formed (15 + 16, 16 + 30, and 14 + 27). The study of all probable groups results in the group that contains blocks 1122 and 23 as the most unstable group (Fig. 2). As has been shown, using the KGM method instead of the KBM method will result in more blocks being considered in the stability analysis, and the KGM method will yield a larger volume of unstable blocks than the KBM method. Stability analysis of the rock mass will lead to the study of groups of blocks, not just a single block. Key Group Method is a clustering technique based on the analysis of all neighboring blocks of a key block that searches for the most unstable key group.



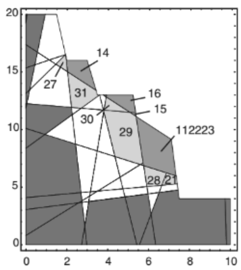
(a) Initial situation of a rock slope
KB 11, 12, 13, 14: stable



(b)Grouping of blocks 12+13
KB 11, 14: stable
KG 1213: unstable



(c) Grouping of blocks 11+22
KB 14, 15, 16: stable
KG 1122: stable



(d)Grouping of blocks 1122+23
KB 14, 15, 16: stable
KG 112223: unstable

Fig. 2-. An example of the key-group method carried out on a fractured rock slope

3- Key-group analysis based on the DDA method

As explained above, a key-group can remain stable during the analysis process and become a large key-group composed of several single blocks. The stability analysis of large key-groups with complex geometry can be complicated and conventional KGM as well as the standard KBM are incapable of analysis of such problems.

The KBM uses a limit-equilibrium analysis and assumes that the blocks are rigid with smooth surfaces. Since a key-group contains individual blocks and includes the possibility of relative movements along their common surfaces, the rigidity assumption within a key-group analysis can cause debating. This problem will become critical when the blocks slide on two or several sliding surfaces.

To solve this problem, we suggest adapting the DDA method for the stability analysis of key-groups in the fractured rock slopes. The capability of using this method has been proved by authors earlier [30]. On this approach, after identifying the key group, its analysis is performed using the Discontinuous Deformation Analysis method. Finally, if the key group was unstable, it will be eliminated according to the algorithm. The search for the most unstable key-group will continue until it finds the critical surface of failure (Fig. 3).

The important point is that although the numerical method of DDA is capable of analyzing rock slope stability, given that a rock slope has a large number of rock blocks, the volume of numerical computations will be large, and at times beyond the computing capacity of the computers. This is especially acute about the real geometry of the joints. In practice, the combination of these two methods will result in numerical computation for part of the cross-section and not all of it, which can experience the most critical state in a stability analysis.

4- The Discontinuous Deformation Analysis (DDA) method

The discontinuous deformation analysis method is an algorithm that was initially introduced by Shi [24] to solve the problems of discontinuous rock mass analysis under different loading conditions. Shortly thereafter, computer programs developed based on this algorithm, which indicated a huge potential for this approach. Various changes have been reported on the original DDA formula in rock mechanics sources [31, 32, 33].

The unique capabilities of this approach include the ability to model large displacements. In this method, the conditions of the blocks in the contact points are updated in an iterative process. This approach changes the condition of the least unrealistic resistance against the movement of the blocks relative to each other. In the following, an overview of the initial formulation provided by Shi is given.

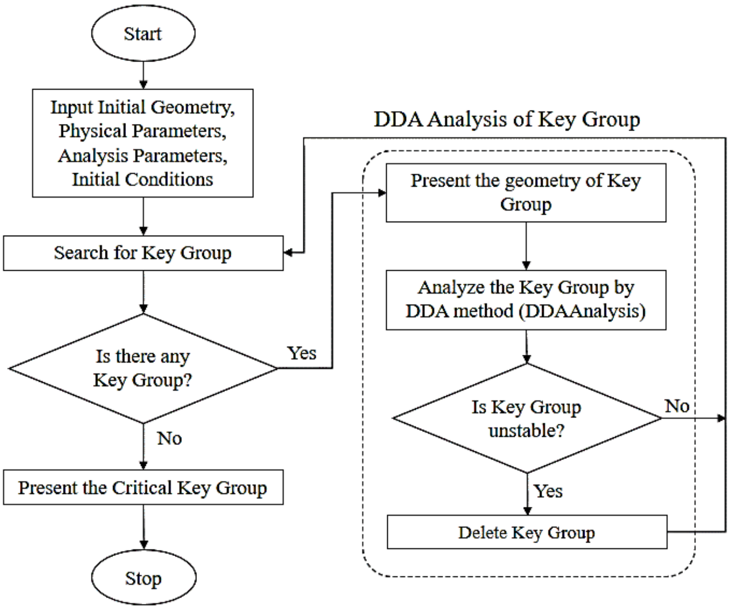


Fig. 3-. The flow chart of the developed method

4-1- Block Deformations

In the initial formulation of the DDA, a first-order polynomial displacement function is employed. By adopting the first-order displacement approximation, The DDA method assumes that generally, stresses and strains are constant on each block.

The displacements (u, v) at desired point (x, y) in a block i in two dimensions can be related to six displacement variables [21, 23]:

$$D_i = (d_{1i} \quad d_{2i} \quad d_{3i} \quad d_{4i} \quad d_{5i} \quad d_{6i})^T = (u_0 \quad v_0 \quad r_0 \quad \varepsilon_x \quad \varepsilon_y \quad \gamma_{xy})^T \tag{1}$$

In which (u_0, v_0) is the rigid body transition at a certain point (x_0, y_0) in the block, r_0 is the angle of rotation of the block with the center of rotation in (x_0, y_0). ε_x, ε_y, γ_{xy} are normal and shear strains in the block. As shown by Shi [21], the complete first-order approximation of the displacement is as follows:

$$\begin{bmatrix} u \\ v \end{bmatrix} = T_i D_i \tag{2}$$

In which

$$T_i = \begin{bmatrix} 1 & 0 & -(y-y_0) & (x-x_0) & 0 & (y-y_0)/2 \\ 0 & 1 & (x-x_0) & 0 & (y-y_0) & (x-x_0)/2 \end{bmatrix} \tag{3}$$

This equation enables calculations of displacements at any point inside the block (in particular, at the corners), when displacements are given at the center of the rotation, and also strains are known (constant on the block). In the formulation of the DDA method, the center of rotation with the coordinates (x_0, y_0) corresponds to the center of the block.

4-2- Minimizing potential energy

According to the second law of thermodynamics, a mechanical system under loading (external and/or internal) must move or be deformed in a direction that minimizes the overall energy of the system. The overall energy includes the potential energy from external loads, system constraints and internal deformations (strain energy) of objects, kinetic energy due to block mass and energy absorbed by the system (dissipated irreversible energy in the system, energy dissipated through friction, and heat generation, for example) [34]. Minimizing the energy of the system will create an equation of motion for the system. In the Finite Element Method (FEM), this is the so-called principle of energy minimization.

Let U_i be the potential energy due to different deformation mechanisms (external loads, strain energy, etc.), K be the kinetic energy and W be the dissipated energy in the system, the total energy Π is given as follows:

$$\prod = \sum(\mathbf{U}_i) + \mathbf{K} + \mathbf{W} \tag{4}$$

The general energy minimization is done by the first-order differential for the displacement vector $\mathbf{d} = \{d\}$ and is written as follows:

$$\frac{\partial \Pi}{\partial \mathbf{d}} = \left[\sum \partial(\mathbf{U}_i) + \partial K + \partial W\right] / \partial\{d\} = 0 \tag{5}$$

Equation 5 gives a weak form of the equilibrium equation describing motion and/or deformation of the block system. Differentiation can be done separately for individual energy mechanisms, and therefore, due to such individual mechanisms, the local equations can be generated [34].

4-3- Equilibrium equations

The first step in minimizing energy is defining the energy Π as $\Pi = F(d_i)$, as a function of the vector of nodal displacement d_i of a block or element i, for a particular energy mechanism. As a result, when only one block (element) *i* is intended, the minimization operator $\partial\Pi/\partial d_i$ will lead to:

$$k_{ii}d_i + f_i = 0 \tag{6}$$

On the other hand, if two blocks *i* and *j* (or two elements *i* and *j* belong to two different blocks) are in contact with each other, the resulting equation will be:

$$\begin{cases}(k_{ii} + k_{ij})d_i + f_i = 0 \\ (k_{ji} + k_{jj})d_j + f_j = 0\end{cases} \tag{7}$$

Then these local equations are coupled together. So that the final and general equations of motion are obtained using the same method used for the finite element method (FEM). In the case of *N* blocks, with m_i main variable in each block (such as displacement), the minimization will lead to the simultaneous equation of block, which is written with the following symbols:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & \cdots & k_{1N} \\ k_{21} & k_{22} & k_{23} & \cdots & k_{2N} \\ k_{31} & k_{32} & k_{33} & \cdots & k_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k_{N1} & k_{N2} & k_{N3} & \cdots & k_{NN} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_N \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_N \end{Bmatrix} \tag{8}$$

Diagonal expressions (*i* = *j*) represent the material properties of each block. For example, they usually contain sub-matrices of elastic deformation and inertia of block *i* (*i* = 1, N). Off-diagonal sub-matrices ($i \neq j$) are described by the state of contact between block *i* and block *j*. Since each block *i* has six degrees of freedom, which are described by the elements of equation (1); each k_{ij} in equation (8) will be a matrix of 6×6. As well as each f_i is also a sub-matrix of 6×1 that describes the loading conditions on the block *i*. Equation 8 can often be written in an express form as follows:

$$KD = F \tag{9}$$

Where K is a $6n \times 6n$ stiffness matrix, and the D and F are displacements and forces $6n \times 1$ matrices, respectively. Overall, the number of unknown displacement parameters is equal to the total degrees of freedom of all blocks. Remarkably, the system of equations (8) is similar to the form of finite element equations. Solving the system of equations (9) will be done by considering a set of inequality associated with the kinematic of the block (for example, no-penetration and no-tension between blocks) and Coulomb's Friction law for sliding along the block edges. To solve the above system of the equations, first, the primary results are checked out to determine how the constraints are satisfied. If tension or penetration is found along any of the contacts, new constraints are applied using hard springs. This process is repeated so that there is no penetration and no tension at the contacts of any blocks. This process has been called as "Open-Close" iteration. Therefore, the displacement variables are obtained during a one-time step process through an iterative process [35].

Stiffness matrices k_{ij} are obtained by minimizing the energy of different mechanisms based on the assumptions about material behavior, loading cases, types of initial boundary conditions, and so on. For example, one of these considerations is discussed in the following. Fig. 4 shows an overview of the DDA method [36].

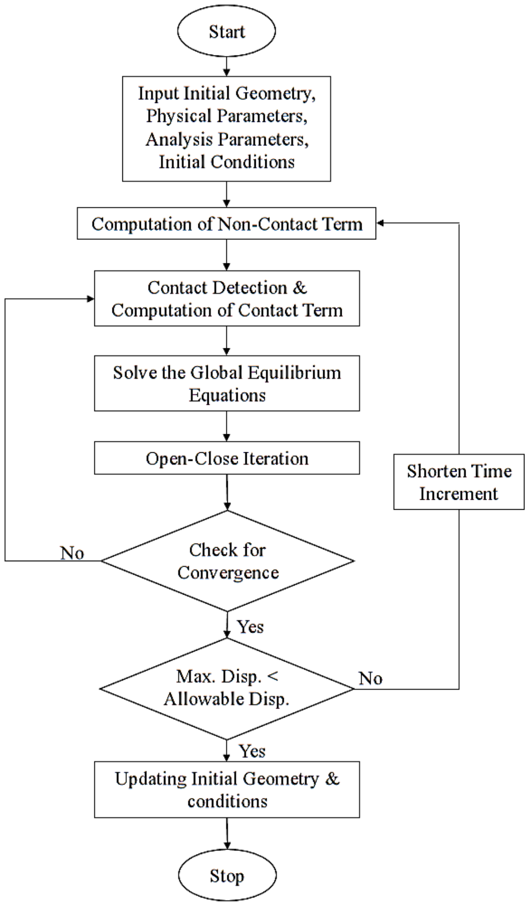


Fig. 4-. Flow chart of DDA program

5- Applications of Key Group analysis by DDA method

For the numerical modeling of the key group, the RAD[1] computer program has been used. The RAD computer code was designed by the authors to analyze the displacements and deformations of the rock masses based on the Discontinuous Deformation Analysis method [36]. This program, using optimized and updated algorithms, allows for analysis with the least kinematic errors. In the developed method, instead of calculating the inter-block forces (similar to SKGM), the key group will be analyzed numerically. One of the most important results of this work is increasing the accuracy of the KGM method and its ability to model the natural geometry of discontinuities.

There are two examples to verify the proposed algorithm implemented in the KGM code. These examples are presented by authors earlier in the paper on the presentation of the SKGM method [18] and were validated by the DEM method.

5-1- Example I: Simple key-group consists of two blocks

The first example is a block system that consists of four blocks. The bottom block is fixed against any movement and the upper block moves freely. Fig. 5 shows the configuration of example I schematically. This simple model provides a more detailed examination of the situations far from undesirable effects.

The analysis of this block system by the method of KGM shows that there is an unstable group consisting of two blocks (Fig. 6). According to the original key group method, the two blocks (blocks 3 and 4) are combined to form a key group. Finally, the results of the stability analysis of this key group will indicate the stability status of the model. But according to the developed method, once the key group has been identified, the blocks will not be combined, and their stability will be assessed using the DDA method. Finally, according to what was said about the original method, the results of the stability analysis of this key group will indicate the stability status of the model.

The properties of the material for all blocks are unique and areas are listed in Tab. 1. The critical friction angle (max value which leads to complete failure) calculated with KGM is equal to 11°. Comparison of the results of the developed method with the results of the key group method and the numerical method of DEM (in [18]) shows that the accuracy of the developed method is acceptable. Figure 6 shows the results of the analysis using methods of KGM and developed KGM respectively.

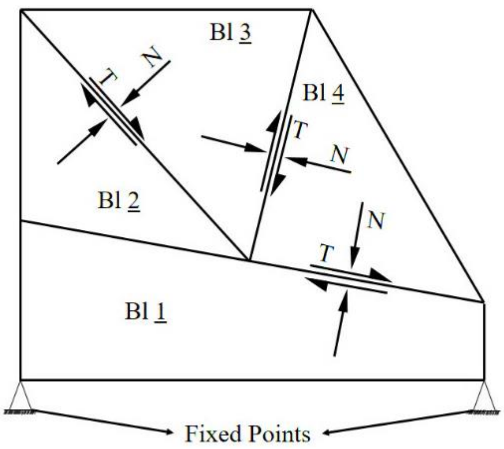


Fig. 5-. Schematic image of example *I*

Table 1-. Properties of the model in example I

Parameter	Value
Cohesion	0.02 MPa
Friction Angle	11°
Poisson Ratio	0.25
Density	2500 Kg/m3
Normal Stiffness	10
Time Interval	0.0013 s
Gravity Acceleration	9.81 m/s2

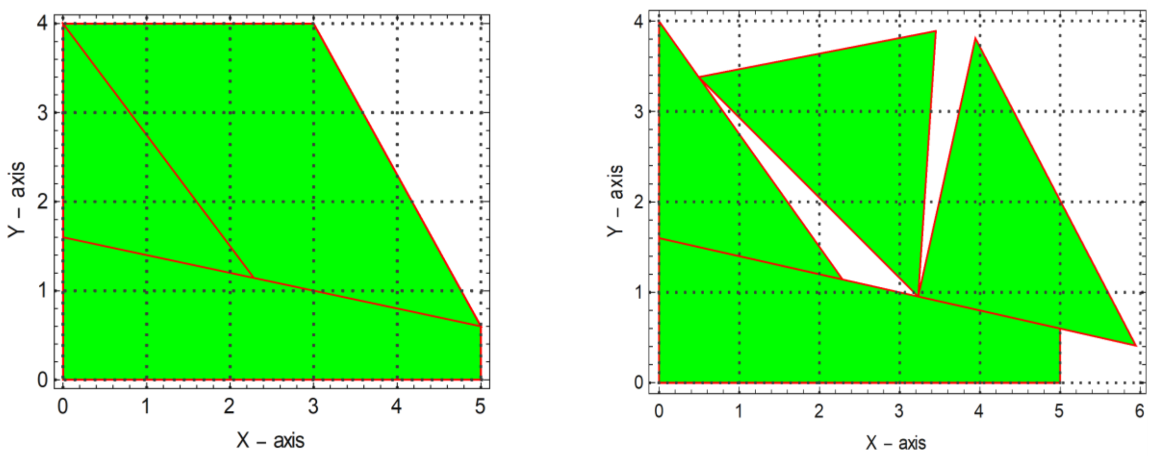


Fig. 6-. Results for (a) original KGM and (b) developed KGM

5-2- Example II: Rock slope analysis

The second example is also a block system that implies a rock slope with a height of 95*m*. The properties of the material for both blocks are unique and are as listed in Tab. 2. Fig. 7 shows the configuration of example II schematically. As said before, in this paper, the aim is analysis the key group by DDA numerical method. So, the overall analysis of the rock slope and choosing the key-group is ignored.

Analyzing this rock slope with the KGM method illustrates the presence of a candidate key group. The studies based on the original KGM method were showed the stability of this key group, whiles analyzing this model by the SKGM method was implied the instability of this key group. The results of numerical modeling in this study validate this instability.

As said before, one of the weaknesses of the Sarma-based KGM method was ignoring the actual geometry of joints in this key group. In the developed method, after choosing a key group, blocks will analyze based on their actual geometry. In a forward example, after identifying the key group, it will analyze with the aim of the DDA method. The result of this analysis will be the basis of decisions about the key group. As it can be seen in Fig. 8, the result of the analysis demonstrates the instability of the key group.

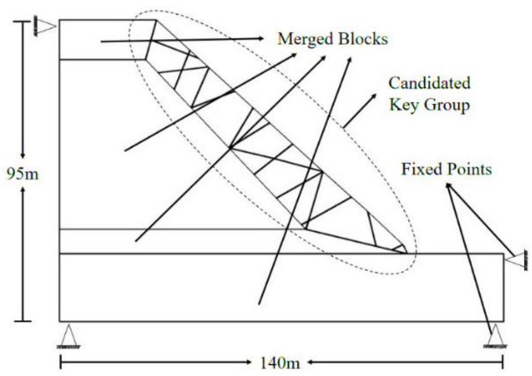


Fig. 7-. Schematic image of example II

Table 2-. Properties of the model in example II

Parameter	Value
Cohesion	0.02 MPa
Friction Angle	22˚
Poisson Ratio	0.25
Density	2500 Kg/m^3
Normal Stiffness	1
Time Interval	0.0013 s
Gravity Acceleration	9.81 m/s^2

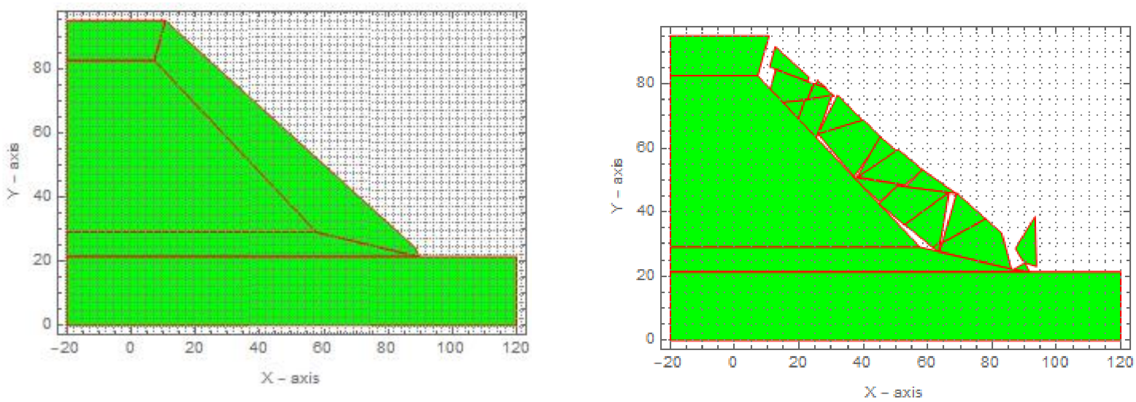


Fig. 8-. Results for (a) original KGM and (b) developed KGM

6- Conclusions

The key group method is an analytical method for analyzing the stability of rock slopes. This method is a fast and accurate solution that proceeds the stability of the jointed rock slopes with a progressive analysis. In this paper, the authors aimed to develop the KGM method using the numerical method of Discontinuous Deformation Analysis for the stability analysis of the candidate key group. For this purpose, in addition to developing a computer program of the DDA method (RAD), the required algorithm was presented and implemented. As a result of this development, capabilities were provided for this method; such as the ability to analyze the candidate key groups using accuracy and capability of the DDA method and enabling the analysis of key groups with real joint geometry.

Comparison of the results of the analysis by the developed method with the results of the original KGM and DEM methods presented in previous studies indicates desirable accuracy and precision of the results and good capability in analyzing the stability of jointed rock slopes with a high number of blocks.

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