Evaluating the Effect of Block Aggregation Approach on Ultimate Pit Limit Characteristics Using the Linear Programming Model

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Abstract

An open-pit mine production planning begins with determining the ultimate pit limit of an open-pit mine. The ultimate pit limit solver selects blocks whose total economic value is maximum while meeting the slope constraints. In other words, a group of blocks that maximize a selected parameter, such as profit, metal content, or net present value, is considered in determining the ultimate pit limit. Also, the ultimate pit limit is designed to select the waste dump location, surface facilities, extractable reserves, and the amount of waste removal. The production planning problem in large-scale open-pit mines is referred to as an NP-hard problem because it cannot be solved in a reasonable computational time. To solve this, various methods, including aggregation methods, have been proposed to reduce the size of the issue. In this paper, to evaluate the efficiency of the block aggregation technique based on the pit values and computational times, at first, the heuristic Tabesh-AskariNasab aggregation algorithm was applied to the block models with 2400 and 11400 blocks. Then the ultimate pit limit based on the original block model and reconstructed block models were determined using the linear programming model. Comparing the results in both block models indicates that the block aggregation approach considerably decreased computational time while generating near-optimal pit values. These results are more critical in large-scale production planning problems, exactly in open pit mine scheduling. Furthermore, the slope of pit walls was decreased by increasing the size of clusters, and the stripping ratio increased in both block models.

1. INTRODUCTION

The Ultimate Pit Limit (UPL) design and the production schedule are two essential parts of the open-pit mine production planning process. The UPL indicates the size and shape of the open-pit mine at the end of its life. Also, with UPL design, the total extractable reserve and the profitability of the mine are determined. The waste dump, processing plant, and other facilities are located after determining the UPL [1].

The UPL is determined by different methods based on the block model. The orebody is divided into blocks and made a block model based on the alignment of exploratory boreholes and the height of the extraction benches. In this case, grades are assigned to each block using geostatistical techniques and borehole grade information to make a grade block model. Finally, with the economical parameters of the studied deposit and grade block model, its economic block model is prepared and based on this model, the UPL is determined [2].

Various methods, such as the Floating Cone algorithm (FC), dynamic programming, and Lerch-Grossmann Algorithm, have been developed to design the optimum UPL. Each of these methods has particular advantages and disadvantages. The FC is the simplest among these methods that Pana introduced in 1965. An upward cone is first designed for ore blocks based on the desired slope angle in this method. Then the value of all the blocks in the cone is added together. If the result is a positive value, all the blocks inside the cone are removed. Otherwise, it is ignored. In this case, other ore blocks are searched, and cones are formed. This process continues until there are no more ore search blocks left. The results of this algorithm depend on the direction of the investigated model. This algorithm cannot provide an accurate answer or a mathematical guarantee of an optimal solution [1]. The Lerch-Grossmann algorithm is the most complex method proposed based on graph theory. It can be mathematically proved that the Lerch-Grossmann algorithm can find the optimum solution, but it takes high computational time [1, 3, 4].

Among different methods presented to design the UPL, mathematical models such as linear programming can generate real optimal solutions. Several researchers have used the mentioned methods to design the UPL of open-pit mines. AskariNasab et al. used the Intelligent Open Pit Simulator (IOPS) to determine the UPL [5]. Sayadi et al. used artificial neural network to determine the UPL [6].

Khodayari proposed a mathematical algorithm theory to design the UPL. This algorithm is formulated as a linear programming problem. In this method, only the positive blocks, which can offset the cost of the negative blocks, are placed in the UPL [7]. Rahimi et al. have proposed a logic-mathematical algorithm based on the design and mining economics parameters to design UPL. This algorithm creates an iterative process among the various design components and directs them to the maximum value of the economical parameters of the mine[8].

In many mathematical-based techniques in large-scale mines, solving the UPL problem is time-consuming because their block model includes many blocks, which increases the number of constraints and decision variables. So, in 1983, Gershon introduced a directional model, in which blocks of a block model are aggregated together in columns or pillars. These columns consist of blocks that are stacked in a column. Each column extends to the bottom of the orebody bed. By applying this method to an actual deposit, the number of integer variables was significantly reduced due to the reduction in the number of blocks compared to the original block model [9].

Ramazan et al. have proposed the fundamental tree algorithm that reduces the number of variables without negatively affecting the model resolution by adopting slope and precedence constraints. The clusters' size and number in this algorithm cannot be controlled [10-12]. AskariNasab et al. reduced the number of decision variables using block aggregation. In this method, the blocks in each bench were aggregated based on the types of rocks, location, and grade distribution in each block using the fuzzy clustering method. These clusters were called mining cuts [13]. In 2011, Tabesh and AskariNasab introduced a two-stage clustering approach to reduce the size of the open pit mine planning problem. The first step uses the agglomerative hierarchical algorithm to cluster the blocks based on a similarity index. In this case, the blocks are classified based on the rocks, mineral grade, and distance between blocks on each bench. In the next step, the tabu search process reduces the arcs between the generated and the lower bench clusters. By performing these two steps, the number of binary decision variables is decreased significantly [14].

Ren and Topal used the fuzzy C-Means cluster algorithm to cluster blocks on mining benches [15]. In 2016, Jelvez et al. proposed a heuristic block aggregation algorithm. The blocks are first aggregated in this algorithm, and then the aggregate problem is solved using integer programming techniques [16]. In 2018, Mai et al. proposed a new block aggregation algorithm called the Top Cone Algorithm. This algorithm aggregates blocks into Top Cones to reduce the number of variables [17]. Lotfian et al. also used block aggregate to solve the open-pit mine planning problem. At first, mother clusters were created by aggregating the blocks by the k-means method. This work was done in a mathematical programming problem and then solved with a Genetic Algorithm (GA) [18].

This paper's main idea is to evaluate the efficiency of the block aggregation technique based on the pit values and computational times. For this reason, the UPL problem was solved using a Linear Programming (LP) model based on the original and aggregated block models. Thus, the Tabesh-AskariNasab clustering algorithm [19] was implemented to aggregate the blocks in two block models with 2400 and 11400 blocks. The heuristic Tabesh-AskariNasab algorithm aggregated the blocks into mining–cuts units. A comparison of the results indicates that the block aggregation approach considerably decreased computation time while generating near-optimal pit values, which is more critical in large-scale production planning problems.

2. LINEAR PROGRAMMING MODEL OF UPL

The design of UPL can be achieved using a LP model based on the economic block model of the deposit with slope constraints. In other words, for each block placed in the UPL, all the blocks found in its extraction cone must be located in the UPL. In addition, the total value of the blocks located in the UPL has been the highest possible value.

Equations 1 and 2 show this mathematically [20, 21].

$$\text{Objective fanction: } Max \sum_{i=1}^{N} x_i v_i \quad (1)$$

$$\text{Sabject to: } x_i \le x_j \quad i = 1,2,,\dots,N \quad \forall j \in P_i \quad x_i \in \{0,1\} \quad (2)$$

Where v_i represents the economic value of block i, and N is the total number of blocks in the block model. x_i is a binary variable for block i. If a block is within the UPL, its value is one; otherwise, it is zero. j is the precedence block of the block i, which means is it must be extracted before the block i, provided that the slope constraint is met. P_i is a set of blocks located in the extraction cone of block i. As the size of the block model (number of blocks) increases, so does the number of variables and decision constraints. As a result, sometimes, the problem cannot be solved logically.

3. TABESH-ASKARINASAB HEURISTIC ALGORITHM

Mathematical models can generate real optimal solutions for open pit mine planning design. However, block models with many blocks because of the large number of decision variables limit the efficiency of these models. To overcome this limitation, Tabesh and AskariNasab presented a heuristic algorithm for block aggregation to make the mathematical models tractable [14, 19]. In this algorithm, the similarity index of blocks is determined to cluster blocks into mining cuts. Therefore, for calculating the similarity indices, some attributes of blocks such as location, grade, and rock type are chosen, and each level of the block model is clustered separately.

Tabesh and Askari-Nasab borrowed the idea of penalty values from Dosea [22] to define the similarity value between block i and block j according to equation (3):

$$s(i,j) = \frac{R_{ij} \times C_{ij}}{ND^{W_D} \times NG^{W_G}} \quad (3)$$

Where $S(i,j)$ is the similarity between blocks i and j. The similarity between each block and itself is set to zero in the similarity matrix. R_{ij} is the similarity index of rock type, calculated by equation (4). If two blocks i and j have different rock types, then R_{ij} takes a penalty value of $r \in [0, 1]$, and if two blocks have the same rock types, R_{ij} takes one. If the rock type is an influential factor in clustering, lower values should be assigned for the r parameter.

C_{ij} is the parameter calculated by equation (5) and checks that blocks i and j are in the same clusters. If two blocks i and j are in different clusters, then C_{ij} takes a penalty value of c ∈ [0, 1], and if two blocks are in the same clusters, C_{ij} takes one.

ND denotes the normalized distance value of blocks determined from equations (6). D_{ij} is the Euclidean distance between the centers of two blocks i and j, calculated from equation (7), and D_{max} is the maximum distance among all blocks at the same level of the block model. In equation (3), the normalized distance factor is powered to W_D which is the weight of the distance factor. A higher value of W_D recompenses other factors and leads to circular clusters.

NG represents the normalized grade difference of two blocks, i and j, calculated by equations (8). G_{ij} is the Euclidean distance between the grade of two blocks calculated from equation (9). G_{max} is the maximum grade difference among all blocks in the current level of the block model. The normalized grade difference is powered to W_G, which is the weight of the grade factor. A higher W_G value makes the grade factor more effective in clustering.

This clustering algorithm is based on an agglomerative pattern, so each block of the current level is first considered a cluster. Assuming N number of blocks in the current level, two N by N adjacent and similarity matrixes are established and iterative according to the similarity index of blocks or pre-formed clusters; similar blocks or clusters are merged to form new clusters.

$$R_{ij} = \begin{cases} 1 & \text{if two blocks are from same rocktype} \\ r \in [0,1] & \text{otherwise} \end{cases} \quad (4)$$

$$C_{ij} = \begin{cases} 1 & \text{if two blocks are at the same cluster} \\ c \in [0,1] & \text{otherwise} \end{cases} \quad (5)$$

$$ND = \frac{D_{ij}}{D_{\max}} \quad (6)$$

$$D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (7)$$

$$NG = \frac{G_{ij}}{G_{\max}} \quad (8)$$

$$G_{ij} = \begin{cases} \sqrt{\sum (G_i - G_j)^2} & \text{if } G_i \ne G_j \\ \varepsilon & \text{if } G_i = G_j \end{cases} \quad (9)$$

After calculating the similarity matrix, the neighborhood matrix is formed in the next step. Neighbor definition for a block means that all blocks at a certain distance from the geometric center of that block or less than it will be considered neighbors. After selecting similar blocks and merging them into larger clusters, the similarity and neighborhood matrices are updated. There are three methods to calculate the similarity between merged blocks and other clusters: a single link, a complete link, and a mean link. The neighborhood matrix is updated in the complete link using the following equation.

$$S_{(i,j),k} = min(S_{ik}, S_{jk}) \quad \forall k \in \{1,2,\dots,N\} \quad k \notin \{i,j\} \tag{10}$$

An integrated cluster will be adjacent to other clusters if at least one of the blocks in one cluster is adjacent to one of the blocks in the other cluster. The neighborhood matrix is then updated using the following equation, and this process continues until all the blocks are examined.

$$A_{(i,j),k} = min(A_{ik}, A_{jk}) \quad \forall k \in \{1,2,\dots,N\} \quad k \notin \{i,j\} \tag{11}$$

4. EVALUATION OF BLOCK AGGREGATION SCHEME

Two examples were considered to evaluate the efficiency of the block aggregation scheme. The evaluation was based on the UPL values and computation time using the LP model.

4.1. Example one

This example constructed a hypothetical 3D fixed block model with six levels containing 2400 blocks. The dimensions of all blocks are 10*10*10 meters. Each block's grade and rock type were identified in the geological block model. The economic block model was constructed from the grade block model using the economic parameters listed in Table 1.

Table 1. Economic parameters to construct synthetic economic block model

Mining cost ($/ton)	Processing cost ($/ton)	metal price ($/kg)	cut-off grade
10000	90000	330	0.3

The Tabesh-AskariNasab clustering algorithm was implemented to aggregate the blocks given the cluster size (the maximum allowable number of blocks in each cluster). In this example, six cluster sizes of 2, 3, 4, 5, 6, and 10 were considered, and six different reconstructed block models with varying sizes of clusters were generated. Then the LP model was implemented in seven scenarios (one original block model and six reconstructed block models with different sizes of the cluster were considered scenarios 1 to 7, respectively), and the results were discussed.

According to the LP model structure, the number of decision variables and constraints for the original block model and six reconstructed block models is mentioned in table 2. Consequently, the number of decision variables and constraints decreased with increasing the cluster size.

Table 2. The number of decision variables and constraints for the original block model and six reconstructed block models

Description	Decision Variable Numbers	Constraint Numbers
Scenario 1	1420	1020
Scenario 2	801	588
Scenario 3	686	483
Scenario 4	473	355
Scenario 5	427	312
Scenario 6	390	278
Scenario 7	256	188

The UPL was designed using the LP model for all scenarios. The UPL values, the number of ore and waste blocks in the UPL, overall stripping ratios, and the LP model's run time for different scenarios are presented in table 3.

Table 3. The results of the LP model for the original block model and different scenarios of the clustered block

Description	UPL value	waste blocks Number	Ore blocks Number	Number of total blocks	Run time (second)	Stripping Ratio
Scenario 1	11382000	913	332	1245	0.77	2.75
Scenario 2	10992800	990	344	1334	0.28	2.9
Scenario 3	10942600	1005	349	1354	0.25	2.9
Scenario 4	10751800	1025	354	1379	0.15	2.9
Scenario 5	10721600	1025	353	1378	0.14	2.9
Scenario 6	10648200	1031	354	1385	0.14	2.91
Scenario 7	10571800	1037	354	1391	0.06	2.93

The UPL value obtained by the LP model based on the original block model is the optimum and reference value. So the pit values based on the block aggregation approach with various cluster sizes were evaluated concerning this reference value. UPL values and run times of different scenarios are compared in figures 1 and 2.

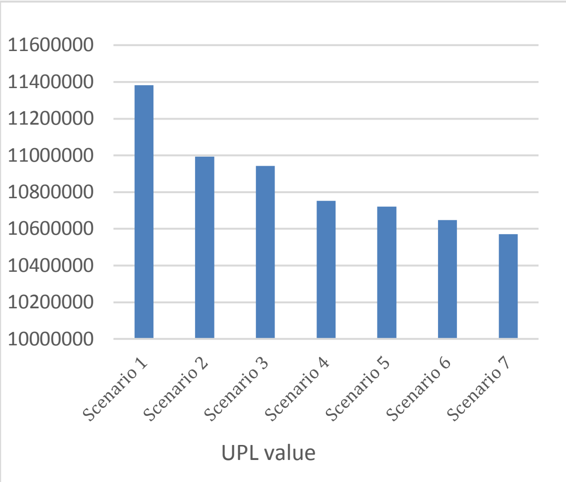

Fig. 1. Comparison of UPL values of different models with the original value.

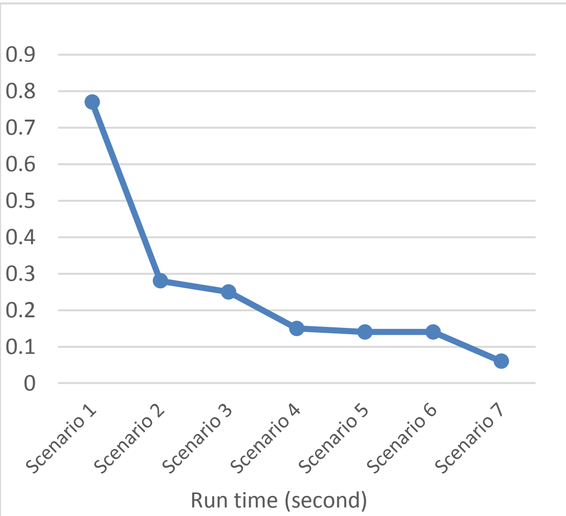

Fig. 2. Comparison of run times of different models with the original value.

The clustering algorithm and the LP model were programmed in MATLAB software and implemented in a system with Intel (R) Core i7 processor specifications - 3.4 GHz CPU and RAM: 16 GB in Windows 7 environment.

Comparison of the results, for example, the cluster size of 3 blocks (scenario 3) decreased the run time by about 67.53 percent. At the same time, its UPL value is only 3.9% lower than the optimum UPL value (reference value). Based on the results shown in Figures 1 and 2, clustering has been done optimally. In addition, the slope of pit walls was reduced in different scenarios, and the stripping ratio increased shown in figure 3.

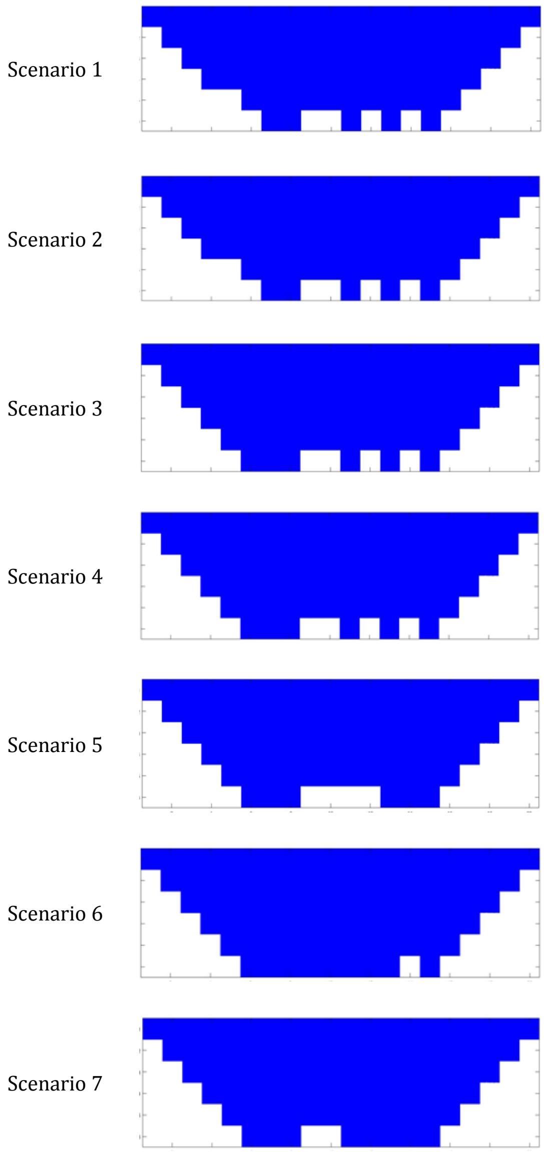

Fig. 3. The section view of the UPL shape of the block model in the east-west direction.

4.2. Example two

The second block model is a real phosphate mine with 11400 blocks and four levels. The dimension of all blocks in this model is 10*10*5 meters. Each block's grade and rock type are identified in the geological block model. The economic block model was constructed from the grade block model using the economic parameters listed in Table 4.

Table 4. Economic parameters to construct synthetic economic block model

Mining cost ($/ton)	Processing cost ($/ton)	sulfate price ($/kg)	cut-off grade
30000	90000	20	0.6

In this example, using the Tabesh-AskariNasab clustering algorithm, seven cluster sizes of 2, 3, 4, 5, 6, 10, and 20 were considered, and seven different reconstructed block models with varying sizes of clusters were generated. The LP model was implemented in eight scenarios. The number of decision variables and constraints for the original block model and seven reconstructed block models is according to table 5. Consequently, the number of decision variables and constraints decreased with increasing the cluster size.

Table 5. The number of decision variables and constraints for the original block model and seven reconstructed block models

Description	Decision Variable Numbers	Constraint Numbers
Scenario 1	10172	7322
Scenario 2	5581	4020
Scenario 3	4646	3339
Scenario 4	2966	2112
Scenario 5	2524	1781
Scenario 6	2109	1468
Scenario 7	1255	854
Scenario 8	629	417

For all scenarios, the UPL was designed using the LP model. The UPL values, the number of ore and waste blocks in the UPL, overall stripping ratios, and the LP model's run times for different scenarios are presented in table 6.

Table 6. The results of the LP model for the original block model and different scenarios of the clustered block

Description	UPL value	waste blocks Number	Ore blocks Number	Number of total blocks	Run time (second)	Stripping Ratio
Scenario 1	76736	1444	5199	6643	17.6	0.28
Scenario 2	76503	1602	5195	6797	8	0.31
Scenario 3	76358	1788	5210	6998	5.5	0.30
Scenario 4	75760	2523	5319	7842	4	0.47
Scenario 5	75550	2779	5362	8141	3.1	0.52
Scenario 6	75419	2894	5333	8227	2.26	0.54
Scenario 7	75198	3173	5391	8564	1.8	0.6
Scenario 8	75040	3376	5440	8816	0.3	0.62

As mentioned earlier, The UPL value obtained by the LP model based on the original block model is the optimum and reference value. So the pit values based on the block aggregation approach with various cluster sizes were evaluated concerning this reference value. UPL values and run times of different scenarios are compared in figures 4 and 5.

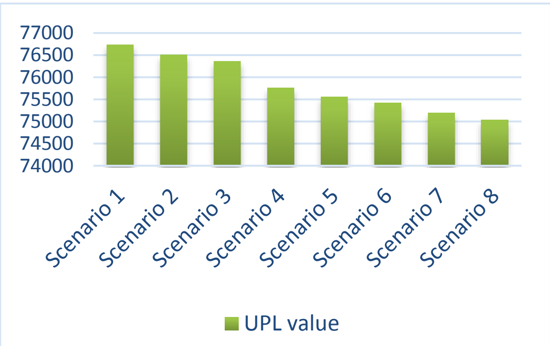

Fig. 4. Comparison of the UPL values of different models with the original value.

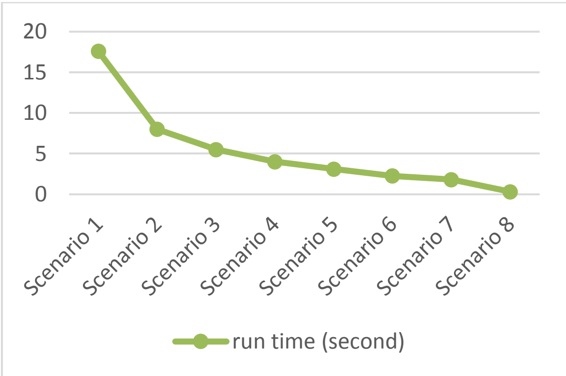

Fig. 5. Comparison of run times of different models with original value.

A comparison of the results indicated that the scenario with the cluster size of 3 blocks decreased the run time by about 68.75 percent. At the same time, its UPL value is only 0.49% lower than the optimum UPL value (reference value). Also, the slope of pit walls was reduced by increasing the size of clusters, and the stripping ratio increased, as shown in Figure 6.

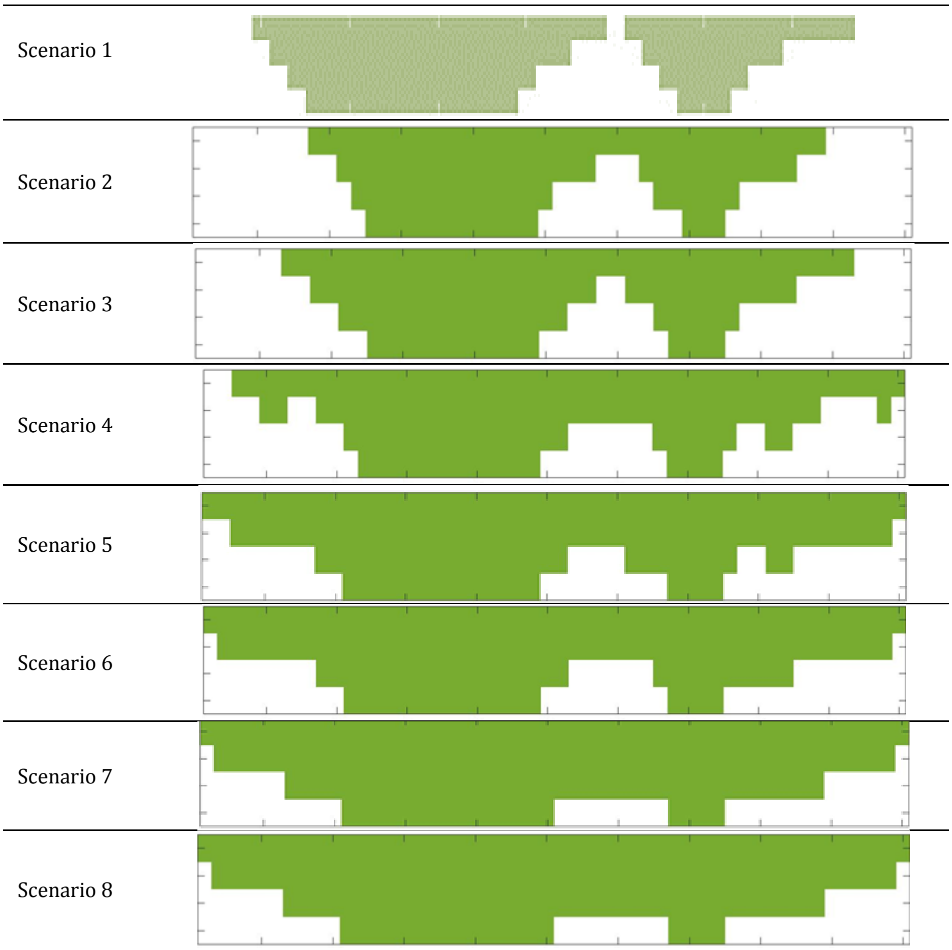

Fig. 6. The section view of the UPL shape of the block model in the east-west direction.

5. CONCLUSION

In this paper, the heuristic aggregation algorithm presented by Tabesh-AskariNasab was used to block aggregation. This algorithm significantly reduced the computation time of the LP model in designing the UPL of open-pit mines, with a decrease in the decision variables. The aggregation algorithm and the LP model were programmed in MATLAB software. Two block models were then used to evaluate the capability of this heuristic method. As a result, the number of decision variables, constraints, and consequently, the computation time in both block models decreased with the increased cluster size. While the computational time reduces significantly with increasing cluster size, these results are more critical in large-scale production planning problems. A comparison of the results indicated that the scenario with the cluster size of 3 blocks decreased the run time by about 67.53 percent, for example, one, and 68.75 percent, for example, two. In this case, at the same time, the UPL values are only 3.9% and 0.49% lower than the optimum UPL value (reference value), respectively. So, an aggregation algorithm can promptly achieve a reasonable answer.

Statements and Declarations

The authors declare no conflict of interest.

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