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Research article

Nearest Linear Failure Surface, a New Method to Determine the Reliability of Numerical Models in Geomechanics

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Monte-Carlo simulation

Abstract

In geomechanical systems, reliability analysis aims to determine the failure probability according to the uncertainties existing in rock mass properties and support materials, as well as diagnosing the significance of each uncertainty. Although there are very diverse methods to determine the reliability of geomechanical models, the employment of precise methods for determining the reliability of a numerical model is practically impossible due to computational difficulties. The only general solution to solve the reliability problem is to use Monte Carlo simulation. However, for most systems, with engineering accuracy, thousands of realizations are required to use Monte Carlo simulation. Although this number of realizations for analytical functions can be performed very quickly, running this number of realizations for a numerical code is practically impossible. In the proposed Nearest Linear Failure Surface method (NLFS), with the least number of runs, the reliability of rock space and its failure probability is investigated during a short period of time and with appropriate accuracy. The idea of the NLFS method is inspired by finding the design point (β-point in the well-known First Order Reliability Method) assuming that the performance function is linear. In this research, a computer code has been developed to implement the NLFS method and by combining this code with FLAC 2D software, reliability of an underground road tunnel with uncertain cohesion, friction angle, and tensile strength of surrounding rock was determined. The results indicate the high efficiency of the proposed method in determining the reliability of numerical models in a very short time and with high accuracy.

1. INTRODUCTION

Recent developments in the field of computer software products and, more importantly, numerical models in geomechanics, have increased the applications of numerical codes in stability analysis of rock spaces. Through these developments, geomechanical engineers have gained the ability to carry out very advanced numerical analyses in a short period and at a low cost. The existing commercial programs for numerical analyses, while being user-friendly, can perform very complicated analyses, such as coupled thermo-elasto-plastic analyses. Despite this, all the input values of numerical models are deterministic, and considering the uncertainties and understanding how and the extent of uncertainties affect is not possible in a simple numerical analysis. Reliability analysis considers the effects of uncertainties on geomechanical stabilities; additionally, it can quantify them and provide more understanding and information than deterministic designs. The main goal of reliability analysis is to determine the failure probability of a system or a model of it.

In geomechanics, unlike the numerical models, reliability analysis of analytical models has been

more considered by researchers [1-8]. For example, Liu et al. (2021), To determine the probability of tunnel face instability, used a novel two-stage parameter estimation method based on a hybrid-evolution Markov chain Monte Carlo (MCMC) algorithm [9]. Using an analytical tunnel face stability model in frictional soil and the inverse first-order reliability method (FORM), Zhang et al. (2024) proposed a new reliability analysis framework [10]. Calculation difficulties in reliability analysis methods in dealing with numerical models can be considered the most important reason for the shortage of reliability analysis of numerical models. Using Monte Carlo simulation with only 50 samplings, You (2005) [11] explored the variability effects of rock strength parameters, with normal distribution, in a FLAC numerical model of a rock tunnel. However, the very few numbers of realizations reduce the accuracy of the proposed risk analysis method. Wile (2006) considered the effect of uncertainties on the results of numerical models for rock pillar stability. In his research, Wile used the results of back analysis and empirical models to assess a representative coefficient of variation, representing the variability of the numerical results [12]. Assigning random characteristics to each numerical element based on random distributions of rock properties has also been common in some research. Griffiths and Fenton [13], dealt with modeling a mine pillar. In their numerical model, they determined the mechanical characteristics of each element according to random distribution of the physical parameters and calculated the failure probability of the mine pillars using the Monte Carlo simulation. Using the Response Surface method, Abdel Massih (2008), investigated the reliability of the numerical model of a soil foundation (using FLAC^{3D} software) [14]. Dadashzadeh et al. (2017) used the response surface method for converting the results of slope stability numerical simulations to an explicit performance function to be used in FORM reliability method. However, RMS+FORM methods are more efficient than simple Monte Carlo simulation, but the accuracy of the method is highly dependent on the evaluated surface curvature near the design point. Using a 3-D finite element limit analysis (FEIA code) and simplified First Order Reliability Method (FORM) Ji et al. (2021) proposed a reliability-based design of tunnel face stability. They also pointed out the computational difficulties of the method. Using Latin Hypercube Sampling (LHS) as a reliability method and a numerical SLIDE2 (Rocscience Inc. 2018) model for rock slope stability impacted by random seismic loading, Zhang et al. (2021) calculated the failure probability of rock slopes considering the spatial variability of cohesion and friction angle. Although the LHS method is more efficient than the simple Monte Carlo simulation method, it is not accurate for determining very small probabilities.

Exploring the above-mentioned research, it is identified that reliability analysis of numerical models has been performed mostly for simple rock structures (often mine pillars), or by applying simple reliability methods with low accuracy. Computational difficulties can be considered as one of the reasons for this issue and another reason is that the accurate solution of reliability for complicated numerical models is very time-consuming. In the new Nearest Linear Failure Surface method (NLFS), with the least number of runs of numerical codes, the reliability of rock space and its failure probability is investigated during a short period and with appropriate accuracy. In this research, a computer code has been developed to implement the NLFS method. By combining the mentioned code with FLAC 4.0 software, the reliability of an underground tunnel whose surrounding rock had uncertain cohesion, friction angle, and tensile strength was determined. The results obtained from the implementation of NLFS method on this numerical model indicate the significance of cohesion and friction angle rather than tensile strength to achieve the intended objective for the designed tunnel.

In this study, first, the reliability problem and its solving approach are investigated, then, the new NLFS method is explained and examined; and finally, the NLFS method is executed on one of the Iranian road tunnel case examples. The results of the implementation of the NLFS method indicate its high capability in determining the failure probability of rock structure design in a short period of time with appropriate accuracy.

2. RELIABILITY PROBLEM

A reliability problem is defined as an uncertain model. The objective of the reliability analysis is to calculate the probability of failure in a system, which can be expressed as [15,16]:

$$p_f = \int_{P<0} f(x_1, x_2, \dots, x_n)\, dx_1\, dx_2 \dots\, dx_n \qquad (1)$$

where p_f, is the probability of failure, $X = (x_1, x_2, \dots, x_n)$ is a random vector of uncertain parameters, $f(X)$ is the multivariate probability density function of the random vector, also known as the joint probability density function and P is the performance function according to Eq. (2):

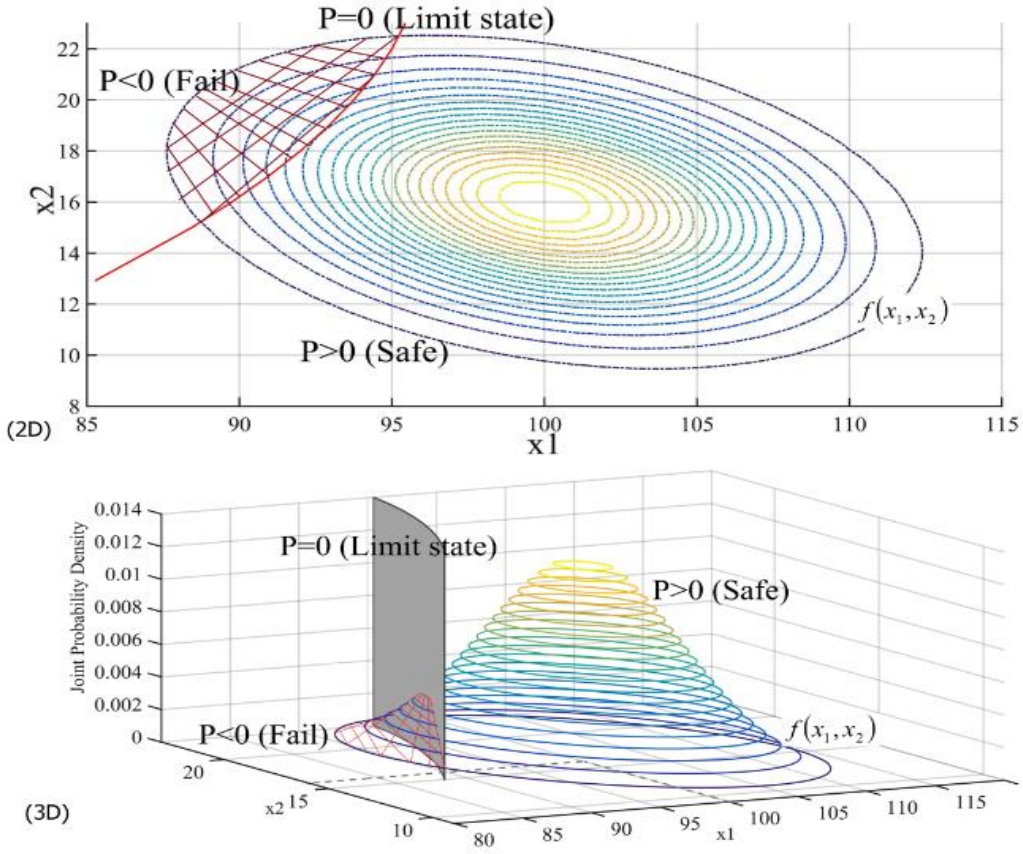

Fig.1. Performance function, probability distribution function, and failure domain of a system with two random variables (x_1 and x_2).

$$P(X)\begin{cases} > 0 & Safe\ state \\ = 0 & Limit\ state \\ < 0 & Failure\ state \end{cases} \quad (2)$$

Fig. 1 shows the probability of failure (semi-infinite domain of integration illustrated by shaded region) and concepts of safe, limit, and failure states of the performance function (for a system with two random variables). In this figure, the shaded area indicates the failure domain and closed ellipses are the contour lines which indicate the multivariate probability distributions of two variables.

In fact, the general problem of reliability (Eq. 1) is a multiple integration that calculates the failure space under a multivariate probability distribution curve of the random parameters. It is noteworthy that, the space volume under each probability distribution curve (univariate or multivariate) is a unit, and a fraction of this volume, representative of the failure area, is equal to the failure probability.

2.1. Reliability Problem Solutions

The only general solution to solve the multiple integrals of Eq. (1) is to use Monte Carlo simulation. Monte Carlo simulation has been established based on repeated samplings from random parameters according to their multivariate probability distribution function and generating the realizations of the performance function. Failure probability is calculated from the ratio between the number of failures, n_f, to the total number of realizations, n, as follows:

$$p_f \approx \frac{n_f}{n} \quad (3)$$

Monte Carlo simulation estimates the failure area by performing numerous random samplings in the total space of random data distribution. The accuracy of this method depends on the vastitude of the failure area and the number of simulations. The expected error of the Monte Carlo simulation depends on the total number of iterations and the real probability of failure. The coefficient of variation of the probability, obtained by Monte Carlo simulation using N-iterations, is calculated as follows [17]:

$$CoV_{pf} = \sqrt{\frac{1-p_f}{p_f \times N}} \quad (4)$$

However, for most systems, with engineering accuracy, thousands of realizations are required in

order to estimate the failure probability using Monte Carlo simulation. Although this number of realizations for analytical performance functions can be performed very quickly, running this number of realizations for a numerical performance code, which takes a lot of time to run each of the realizations, is practically impossible.

Since utilizing a general Monte Carlo solution to determine the reliability of numerical models of rock structures is impossible, using simpler and more optimal methods for solving the reliability problem is considered. One of the most efficient and precise methods for solving reliability problems is the First Order Reliability Method (FORM). The analytical solution for Eq. (1) is not available unless: (I) the multivariable density function is normal, and (II) the performance function is linear or quadratic [18]. These conditions led to the idea of the FORM. Therefore, to solve this problem based on the FORM solution, a probability-preserving transformation, $X = g(Z)$, is assumed so that $Z = (z_1, z_2, \dots, z_n)$ is an independent standard normal vector (each member has a normal distribution with zero mean and unit standard deviation). By assuming this transformation, the performance function can be transmitted to standard normal space spanned by standard normal variables, $(z_1, z_2, \dots, z_n)$, as shown in Fig. 2. In this case, the integral of Eq. (1) is rewritten as [16]:

$$p_f = \int_{P(X)<0} f(X)dX = \int_{P(g(Z))<0} \varphi(Z)dZ \qquad (5)$$

where, $\varphi(Z)$ is the $n-$dimensional standard normal density function [19]:

$$\varphi(Z) = \frac{1}{\sqrt{(2\pi)^n}} \exp(-0.5Z^T Z) \qquad (6)$$

The most likely failure point in the standard space is called the design point or β-point (Z^* in Fig. 2). By replacement of the actual limit state ($P = 0$) with the approximate linear limit state function at the design point ($P_L = 0$), the probability of failure could be approximated by [15]:

$$p_f \approx \Phi(-\beta) \qquad (7)$$

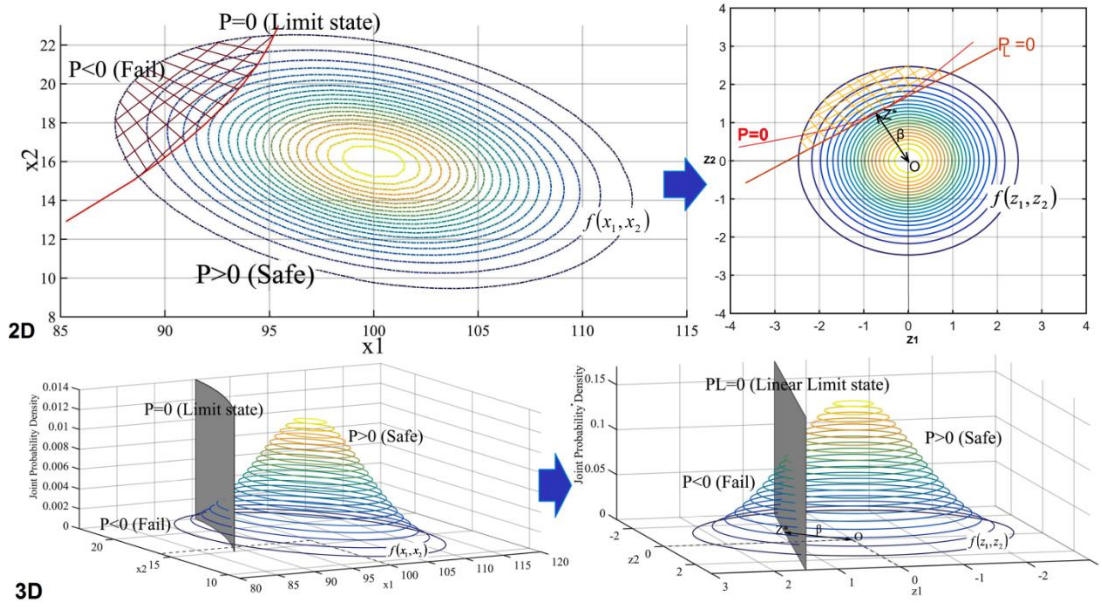

Fig. 2. Transformation of performance function.

where, β is the distance between the origin and design point in normal space ($\beta = \|Z^*\|$) and Φ is the Cumulative Distribution Function (CDF) of standard normal variables. Hence, in FORM solution, the multiple integration of Eq. (1) reduces to a constrained non-linear optimization problem as:

$$\begin{aligned} &\beta = min\sqrt{ZZ^T} \\ &\text{S.t.} \\ &\quad P\big(g(Z)\big) = G(Z) \le 0, \quad X = g(Z) \end{aligned} \qquad (8)$$

where, Z is so-called vector of standard normal random variables, Z^T is the transpose of Z, and g is the translation function. The general form of translation function for non-correlated random variables is defined as [20]:

$$x_i = F_i^{-1}[\Phi(z_i)] \qquad (9)$$

where, each non-normal component, x_i, can follow any arbitrary Cumulative Distribution Function (CDF), F_i.

It is obvious that solving the non-linear optimization problem of Eq. (8) is much easier than the multiple integrations of Eq. (1). Despite this, for numerical models it is not possible to define the performance function (P) analytically. Merely, by running the numerical model for a network of physical random variables, X, discrete points of the performance function can be obtained and a discrete function of the performance and or an approximate analytic performance function can be achieved. Many of the computerized optimization algorithms such as the interior point algorithm [21] also, solve the optimization problem of Eq. (8) in a similar manner. These algorithms depending on the complexity of the performance function, distribution of random variables, and the accuracy required for optimization, carry out some runs of the numerical code. Researchers' experience of this paper implies that, for typical geomechanical problems, the number of runs in order to achieve the optimal answer for Eq. (8) should be between tens to hundreds. Although the number of runs needed in this method is much less than the Monte Carlo solution, the calculation volume and therefore the solution time for most numerical models are still very high.

3. NEAREST LINEAR FAILURE SURFACE

The accuracy of FORM depends on two main factors: (a) the number of design points and (b) the curvature of transformed performance function at the design points [18]. Rackwitz [16] applied FORM method to numerous engineering systems. He observed that, for more than 90 percent of the actual systems, FORM method fulfills all practical needs and the accuracy of FORM is more than sufficient. The reliability analysis of analytical models for rock space stability using FORM method and comparing it with general Monte Carlo simulation and second-order reliability method (SORM) also implies that the accuracy of FORM method is usually very high, especially for performances that are monotone functions of random variables.

According to the high accuracy of FORM method, it can be assumed that the actual failure border in standard normal space ($P = 0$ in Fig. 2) is very close to the approximate linear failure border ($P_L = 0$ in Fig. 2). Thus, the performance could be approximated with a linear polynomial precisely and the performance contours ($P = cte$) should be also near to linear state (Schematic Fig. 6). The mentioned assumption constitutes the basis of NLFS method. NLFS method aims to find the β −point after assuming that the performance function in standard normal space is linear. The implementation steps of the proposed method are presented in Fig. 3 and the flowchart of proposed NLFS algorithm is shown in Fig. 4.

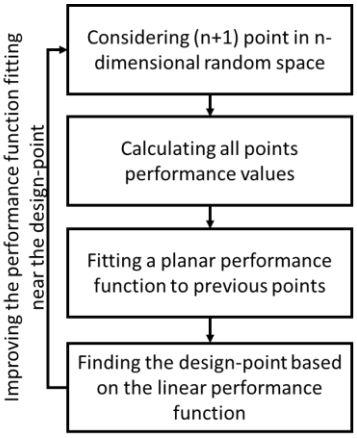

Fig. 3. Implementation steps of NLFS method.

Solution steps of NLFS to determine the reliability of numerical model with n −random variables are as follows:

Step 1) Guessing an approximate design point (β − point): $Z^0 = [z_1^0, z_2^0, \cdots, z_n^0]$, For example, this point can be assumed as the origin of coordinates (mean value): $Z^0 = [0,0, \cdots, 0]$

Step 2) Considering n new points as: $Z^1 = [z_1^0 + k_1, z_2^0, \cdots, z_n^0]$, $Z^2 = [z_1^0, z_2^0 + k_2, \cdots, z_n^0]$,..., $Z^n = [z_1^0, z_2^0+, \cdots, z_n^0 + k_n]$, in which k_i are real numbers opposite to zero. In many situations, considering the value of 1 for k_i is suitable; but in some situations, different values must be considered for k.

Step 3) calculating the performance values for all $n + 1$ previous point using the transformation function and the numerical code: $P_0 = G(Z^0), P_1 = G(Z^1), \cdots, P_n = G(Z^n)$

Step 4) finding the equation coefficients of linear performance function by solving a linear system of equations:

$$M_{n+1\times n+1}A_{n+1\times 1} = P_{n+1\times 1} \qquad (10)$$

which, M is a $n + 1 \times n + 1$ matrix with i^{th} row is given by $(z_1^i, z_2^i, \cdots, z_n^i, 1)$ or i^{th} considered point coordinates obtained in steps 1 and 2. A is an $n + 1$ column vector containing the unknown coefficients of the linear polynomial equation of performance function in n −dimensional normal space: $(a_1, a_2, \cdots, a_n, a)^T$. P is the $n + 1$ a column vector of performances calculated in step 3. Eq. (10) is a linear system with widely available solutions. By solving this linear system of equations and finding the vector A, the linear limit state surface is approximated as:

$$L(Z) = a_1 z_1 + a_2 z_2 + \cdots a_n z_n + a = 0 \qquad (11)$$

Step 5) finding the new β-point (the nearest point on the approximated failure surface to the center of the coordinates) based on geometric rules:

$$Z^0 = \frac{a}{||[a_1, a_2, \cdots, a_n]||^2}(a_1, a_2, \cdots, a_n) \qquad (12)$$

Step 6) repeating steps 2 to 5 until convergence is reached: the actual value of the performance function at the estimated $\beta-$point can be regarded as the error of the $\beta-$point estimation ($err = G(Z^0)$); and calculations will be continued until the error reaches to an acceptable level.

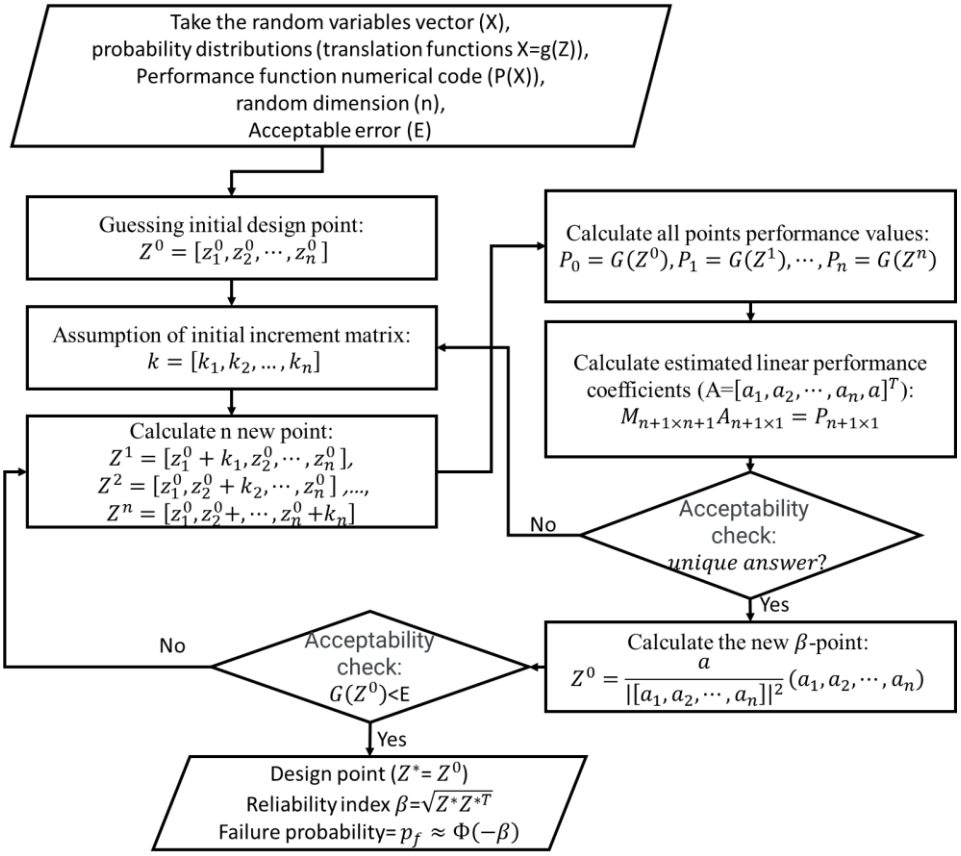

Fig. 4. The flowchart of proposed NLFS algorithm.

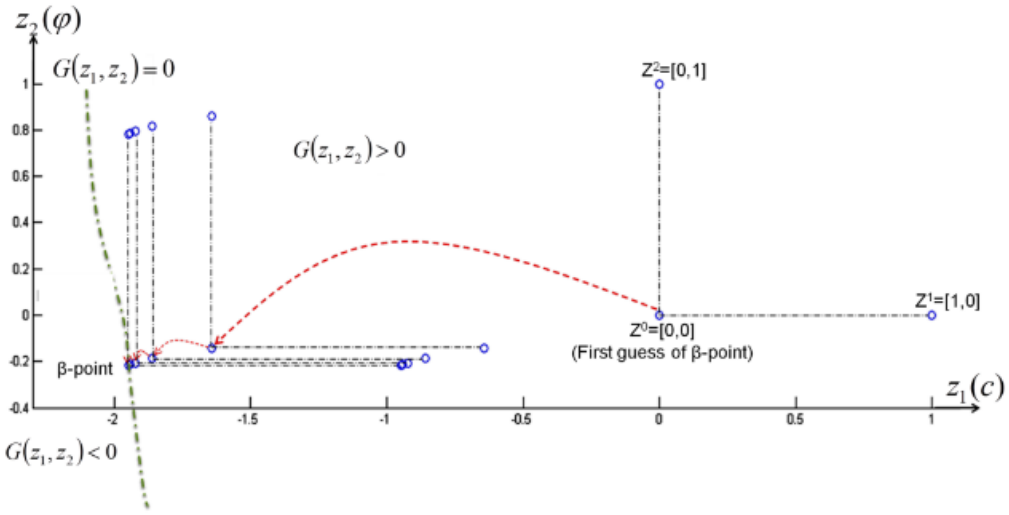

Fig. 5. NLFS loops to find the β-point of an uncertain rock block.

In Fig. 5, the stages of NLFS method until arriving at the β-point and determining the reliability of an actual rock block stability with two random variables of joint cohesion and joint friction angle have been illustrated. In the solution

shown in Fig. 5, all of the k values are considered as equal to 1 and the number of performance function runs is much less than which needed in other reliability methods.

In NLFS method, k values should be selected so that a unique answer would be obtained for the linear system of equations (Eq. 10). In other words, in each cycle of NLFS solution, none of the performance values from n+1 assumed point should be the same as each other's. Otherwise, k values should be changed. To optimize NLFS solution, k values can be considered proportionate to the error value (err). In this state, by approaching the solution to the actual β-point, local tendencies of the performance would affect the calculations and the effect of the far field tendencies of the performance would be reduced, Fig. 6.

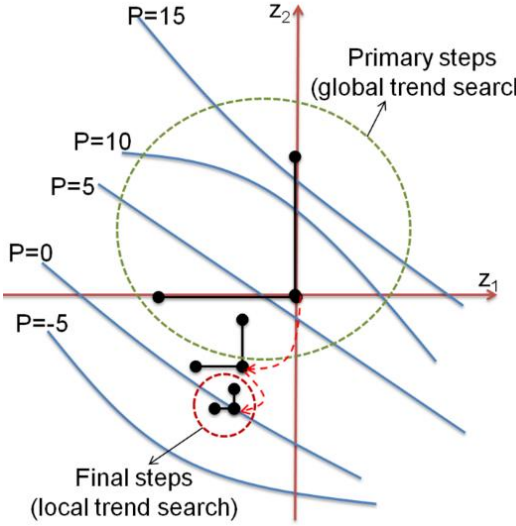

Fig. 6. NLFS optimization, by assuming k values proportionate to the error value.

Fig. 7 illustrates the steps of NLFS method with k values equal to "$2\Phi(err) - 1$" for the same example of Fig. 5. It can be observed that by relating the k values to the error value, convergence to the final answer occurs rapidly. However, it should be noted that due to the simplifications made in numerical models, considering very small values for k is not possible. This is because the possibility of losing the uniqueness of the answer for the Eq. 10.

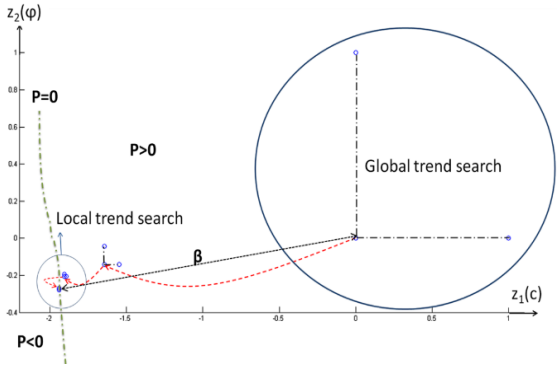

Fig. 7. Optimized NLFS solution considering decreasing values for k.

Sometimes it is preferable to solve for the unknown coefficients of Eq. (10) using regression of all previous loop's points (e.g. m points). For this purpose, Eq. (10) must be rewritten as follows:

$$(M^T M)A = M^T P \qquad (13)$$

in which, M is a m×n+1 matrix with ith row given by $(z_1^i, z_2^i, \cdots, z_n^i, 1)$, M^T is the transpose of M, A is same as the Eq. (10) and P is a m×1 column vector of points performances calculated in previous steps. Note that Eq. (13) is a linear system amenable to fast solutions as well.

3.1. Case example

A horseshoe-shaped tunnel has been excavated deep in the earth, in a place where the vertical stress (maximum stress) is 19 MPa and the horizontal stress (minimum stress) is 15 MPa. The shape and geometrical dimensions of this tunnel have been illustrated in Fig. 8 and the characteristics of the rock mass surrounding the tunnel have been provided in Table 1.

Table 1. Properties of surrounding rock mass.

Deterministic parameters	value	Probabilistic parameters	Probability distribution (PDF)
density	3600 t/m3	Cohesion	$Truncated\ Normal\ distribution$ $(\mu = 0.9, \sigma = 0.15)\ MPa\ > 0$
Bulk modulus	3.2×109 pa	Friction	$Log\ Normal\ distribution$ $(\mu = 35°, CoV = 8\%)$
Shear modulus	1.5×109 pa	Tension	$Uniform\ distribution$ $[0.02 - 0.03]\ MPa$

μ: theoretical mean σ: theoretical standard deviation CoV: Coefficient of Variation

According to the expected operation for this tunnel, the aim for tunnel design is to maintain the convergence of the roof and floor limited to the permissible level of 12 cm. In order to investigate the tunnel's mechanical behavior, a numerical model has been prepared using FLAC 4.0 software. The network of elements around the tunnel space, the investigated monitored points located in the tunnel roof, floor, and walls, and also the tunnel's boundary conditions have been shown in Fig. 8.

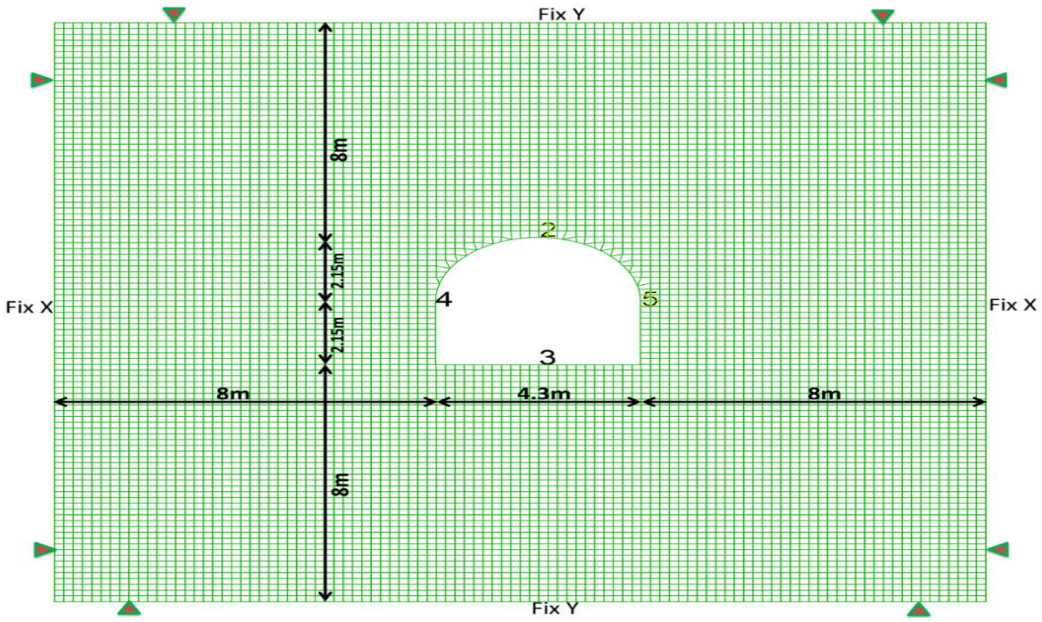

Fig. 8. Tunnel geometry, element network schematic and the boundary conditions of numerical model prepared using FLAC 4.0 software.

3.1.1 Deterministic model

If the tunnel is considered without support, and a deterministic model with the mean values of random parameters ($\sigma_t = 0.025\ MPa$ and $\varphi = 35°$ and $c = 0.9\ MPa$) is provided and then run, final displacement values for the four investigated points on the roof, floor, left and right walls of the tunnel (points 2, 3, 4, and 5 in Fig. 6) will be obtained according to Table 2.

Table 2. Tunnel convergence values considering the deterministic model with average values of random parameters

Monitored point	convergence
Roof (point 2)	2.48 cm
Floor (point 3)	7.48 cm
Right wall (point 4)	3.66 cm
Left wall (point 5)	3.55 cm

With regard to the aim of tunnel designing, performance function of the tunnel can be defined as the Eq. (14).

$$P = 12 - (d_3 - d_2) \qquad (14)$$

where, d_2 and d_3 are considered as the displacement of the points on the roof and floor of the assumed tunnel in y-axis direction (up) respectively and the constant 12cm is the maximum permissible roof-floor convergence. According to the values of Table 2, the tunnel's performance value without support will be equal to $P = 2.04\ cm$. Since the performance value in the deterministic solution is positive, the assumed tunnel (without support) is identified to be appropriate in order to achieve the design purpose (Eq. 2).

3.1.2 Reliability analysis using NLFS Method

In order to solve problems using NLFS method, a MATLAB computerized code has been prepared. Combining this code with FLAC 4.0 software, probabilistic solution of the above-mentioned example in a fully automatic way has become possible. The basis for NLFS computerized code is running the steps 1 to 6 of the NLFS method on every input performance function. For probabilistic solution of the proposed example, tunnel performance function was considered according to Eq. (14). On the first cycle of NLFS, the β-point was considered as $Z^0 = [0,0,0]$ and k value for each random variable is assumed equal to 1. Also, convergence condition of the problem is considered as: $err = G(Z^0) < 0.05\ cm$. By running NLFS code for the mentioned example, the expected convergence condition is met during 9 calculation cycles. Final results for solving NLFS have been obtained according to Table 3.

Table 3. NLFS results for tunnel numerical model

Final β-point (Z*)	[z(C) = −1.666, z(φ) = −1.014 , z(Ten) = 0.005]
Physical β-point (X*)	[C = 0.66MPa, φ = 32.17°, Ten = 0.025MPa]
err = G(Z*)	+0.02 cm
Reliability index (β)	1.9458
Probability of failure (p_f)	2.58 %
No. of loops	9
No. of FLAC model runs	37
Calculation Time	2700 sec

Therefore, the failure probability for the investigated tunnel design is equal to 2.58%. The calculations' run time for this case is a fraction of an hour, whilst for the simulation using the Monte Carlo method, with acceptable engineering accuracy, thousands of iterations of the numerical model runs and hundreds of hours are required. For comparison, the estimated failure probability obtained by Monte Carlo simulation with only 37 realizations (same as the proposed model), based on Equation 4, has a coefficient of variation of 1.0102 or a standard deviation of 0.02606 (2.606%). In other words, Monte Carlo simulation with only 37 realization estimates the failure probability between (0%-10.4%) with an accuracy of 99.73% or between (0%-7.79%) with an accuracy of 95.45% based on the normal distribution properties. Therefore, the number of 37 Monte-Carlo realizations is completely insufficient, and estimated values, with only 37 realizations, are completely meaningless.

Design-point (β-point) obtained by this method provides much engineering information about the rock space status. Every component of the β-point in standard normal space represents the extent and the type of each random variable impacts on the performance of the rock space. The magnitude of each component of the β-point (regardless of its sign) represents the extent of its effect on the efficient performance of the rock space system. Therefore, in the case example discussed here, the cohesion of the rock mass had the highest impact on efficient performance or non-occurrence of unauthorized convergence of the tunnel. The internal friction angle of the rock mass is the second important factor in tunnel stability, but the tensile strength of the rock mass had very insignificant effect on the proper performance of the tunnel. Simply put, in the present example, the reason why the reliability index became large and as a result the failure probability for the performance function of the tunnel (Eq. 14) became small is in the first order due to the probability distribution of rock mass cohesion and in the second order is because of the friction angle pdf.

4. CONCLUSION

In the present research, a new method (Nearest Linear Failure Surface, NLFS) has been provided for reliability analysis and probability investigation for numerical models in geomechanics. The new method can be generalized for every numerical model with different physical variables especially those that have positive or negative monotone effects on the performance function. In other words, this method is more suitable for performances that are monotone functions of random variables (e.g. mechanical properties of rock support and water pressure in rock stability). On the contrary, it is not very suitable for performances that are non-monotone functions of random variables (e.g. joint orientation in rock block stability).

The proposed method, in addition to numerical models, is also significantly useful for highly complex performance functions and reduces the reliability analysis time to a great extent.

The presented method is a very straightforward solution for the First Order Reliability Method (FORM) which has been underpinned by assuming the performance function to be linear in standard normal space. This method achieves an estimated design point (β-point) by running n+1 samples of the numerical code, and improves β-point approximations by repeating the solution chain.

The probabilistic stability of a horseshoe-shaped tunnel was analyzed using NLFS method. Solution results represent the failure probability of this tunnel and the effect of each random variable on the tunnel performance. The solution time was a fraction of an hour while determining the precise failure probability by Monte Carlo simulation requires tens or hundreds of hours.

Although the presented method and other existing reliability analysis methods carry out the probability investigation of a model considering the effect of physical uncertainties, these methods cannot consider the effect of the simplifications and uncertainties existing inherent in numerical models. In other words, simplifications and uncertainties existing in a numerical model are the common weaknesses of both deterministic and probabilistic analyses.

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