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Research article

Probabilistic Pit Limit Design with Different Levels of Managerial Risk Tolerance, Considering the Uncertainty of Ore Grade and Final Product Price (Case Study: Afghanistan's Ainak Copper Mine)

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First Order Reliability Method
Managerial risk tolerance

English Extended Abstract

Summary

The ultimate pit limit actually shows pit location, size, shape, and depth at the end of its working life. Although most scientific research determines the final pit limit by assuming constant design parameters, the existence of design uncertainties leads to significant deviations between expected design results and actual implementation during mining. Two of the most important uncertainties in mining are the grade of the mineral and the price of the final product, which, considering the effect of these uncertainties in the design of the final pit limit, will lead to a more appropriate understanding of the probability of achieving various mining goals. In this research, the pit limit of the Ainak copper mine in Afghanistan has been investigated, assuming the simultaneous uncertainty of the product price and ore grade. by using the exploration data, the distribution of the grade of the ore mineral in different blocks has been determined, and then the historical copper price is investigated, and a lognormal distribution function has been assumed for it. Taking into consideration the price and grade distributions, 9 different pits were designed. In each of these pits, the copper price and mine blocks' ore grades have been assumed optimistically, logically, and pessimistically based on their probability distribution function. Then, by using the First-Order Reliability Method (FORM) and assuming a profit goal for mine pits, the success or failure probability of each designed pit in achieving management objectives was calculated. The results of this research show that although the logical pit has an expected profit of 28 billion dollars, for example, for achieving a profit of 10 billion dollars for a risk-averse manager, using the pessimistic price-logical grade pit (pit number 2) has the highest success probability (91.8 percent). In addition, for a risk-taking manager, reaching the profit goal of 40 billion dollars using the optimistic price-logical grade pit (pit no. 8) has the highest success probability (31.34 percent). The results show that, for different managerial risk tolerances, it is needed to design different final pit limits based on the highest probability of success in managerial goals, and a specific design is not sufficient for different managers.

Introduction

The design of the final open pit limit is usually carried out to maximize the profit from mining and assumes that the design parameters are certain [2]. However, uncertainties in mining can lead to significant deviations in both design parameters and results during implementation [1]. Also, in ordinary final pit limit design methods, the management objectives as well as the different managerial risk tolerance level and their effects on optimum final pit limit are not taken into account [3-13]. In this study, by presenting an efficient algorithm,

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appropriate final pit limits with the highest probability of success in achieving management goals have been suggested for managers with different levels of risk tolerance. It is noteworthy that for different management goals, the suggested pit limits will be very different.

In this study, to consider price as a random variable, the historical price of copper metal over the past 10 years, between 2013-2023 was obtained in terms of $/ton. To eliminate the inflationary effects of the money value, the Producer Price Index (PPI) was used, and the historical copper prices were updated in terms of the dollar value of 2023 Finally, assuming a log-normal distribution for the price, the mean value and standard deviation of the logarithmic price were calculated according to the data. To analyze the uncertainty of the ore blocks' grade, the copper grade was also considered an uncertain parameter with a normal distribution, and its mean and variance were calculated using the Kriging estimation method in the Surpac software. Based on the uncertainty of the block grades as well as the uncertainty of the copper price, 9 pit limits with different grade and price realizations (optimistic, logical, and pessimistic) were obtained based on the Lerch and Grossman optimization algorithm. In this study, the optimistic cases are considered as the design pits with the mean value of the price or grade plus one time the standard deviation, while the pessimistic cases are considered as the design pits with the mean values minus one time the standard deviation, and the logical case is designed considering the mean values of these parameters. Thus, by entering the 9 different grades and price data into the design algorithm, 9 different hypothetical pits are obtained, and these pit limits will be valid for the occurrence of optimistic to pessimistic conditions. Next, using the First Order Reliability Method (FORM) analysis and assuming multiple managerial goals for the expected profit of mining, the probability of their success or failure is analyzed and discussed, and a suitable final pit limit is suggested for risk-averse to risk-tolerant managers. As a result of this research, a final pit limit will be suggested for each management goal that has the highest probability of success in achieving that goal, and for each manager with a specific target level, the results will be different from those suggested by other managers.

Methodology and Approaches

The method used in this research includes the following steps:

1- Estimation of the probability density function of the final mine product price and its translation function to the standard normal space.
2- Assuming a normal probability distribution for mineral grade and estimating the mean and variance of grade for each block with the Kriging estimator
3- Assuming price and block grades translation function to the standard normal space as eq. (1,2) respectively:

$$Y = \sigma_y . Z_1 + \mu_y \quad (1)$$
$$p = exp(y)$$

$$g_i = \mu_{gi} + \sigma_{gi} . Z_2 \quad (2)$$

In which Y is the logarithmic price with logarithmic mean of μ_y and logarithmic standard deviation of σ_y, g_i is ith block estimation grade with mean of μ_{gi} and standard deviation of σ_{gi} and Z1 , Z2 are two independent standard normal random variables.

4- Assuming logical price and grades as $(\mu_y, \mu_{gi}\ \ \forall i)$ respectively, optimistic price and grades as $(\mu_y + \sigma_y,\ \mu_{gi} + \sigma_{gi}\ \forall i)$ respectively and pessimistic price and grades as $(\mu_y - \sigma_y,\ \mu_{gi} - \sigma_{gi}\ \forall i)$ respectively.
5- Design 9 ultimate pit limits with all combinations of assumed (logical, optimistic, and pessimistic) prices and ore block grades.
6- Considering different managerial goals for risk-averse to risk-tolerant managers and defining different performance functions for each goal as eq. 3:

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$$P(X) = P(p,g) = profit - goal = \sum_{i=1}^{I} BEV_i(p,g) - goal \quad (3)$$

In which, X=[p,g] is a random vector of price and blocks grade and $BEV_i(p,g)$ is ith block economic value.

7- Estimating the reliability index and the failure probability of each design pits to reach each managerial goals using the concept of First Order Reliability Method (FORM) as follows [14-16]:

$$\beta = min\sqrt{Z_1^2 + Z_2^2 + \cdots + Z_n^2} = min\sqrt{ZZ^T} \quad (4)$$
$$subject\ to$$
$$P(g(Z)) = 0$$
$$X = g(Z)$$

$$p_f \approx \Phi(-\beta) \quad (5)$$

In which, β is reliability index obtained by optimization formulation of eq. (4), X=[p,g] is the random vector of price and blocks grade, Z=[Z1,Z2] is the random vector of standard normal variables, g is translation function introduced in eqs. (1,2) and Φ is standard normal cumulative distribution function.

8- Suggesting an optimal pit for each managerial objective based on the highest probability of success in the first stage and the highest expected profit in the second stage.

Results and Conclusions

This study obtained nine distinct ultimate pit limits (UPLs) for the Ainak copper mine, considering all combinations of optimistic, logical, and pessimistic assumptions for both prices and block grades. Given the average (expected) profit of $28 billion in this mine (for the logical pit with average price and grades), targets of 10, 20, 30, and 40 billion dollars have been considered for risk-averse to risk-tolerant managers.

The results of implementing the FORM method for each of the four management objectives are shown in Figure (1). In Figure (1), the probability of not achieving the minimum target profit is introduced with the probability of failure. This probability has been calculated for all profit targets and also for all nine pits. According to Figure (1), for a risk-averse manager with a profit target of $10 billion, the lowest risk pit is the pessimistic price pits with a probability of failure of 8.19%. The probability of failure of these pits is almost neutral with respect to grade changes. Therefore, the risk-averse manager should choose the one with the highest expected profit (Pit No. 2). Also, for a logical (risk-neutral) manager with a profit target of $30 billion, Pit No. 5 has the lowest failure probability of 47.13%. For a risk-averse manager with a profit target of $40 billion, optimistic price pits have the lowest failure probability of approximately 68.66%. The failure probability of these pits is also neutral to changes in the block grades.

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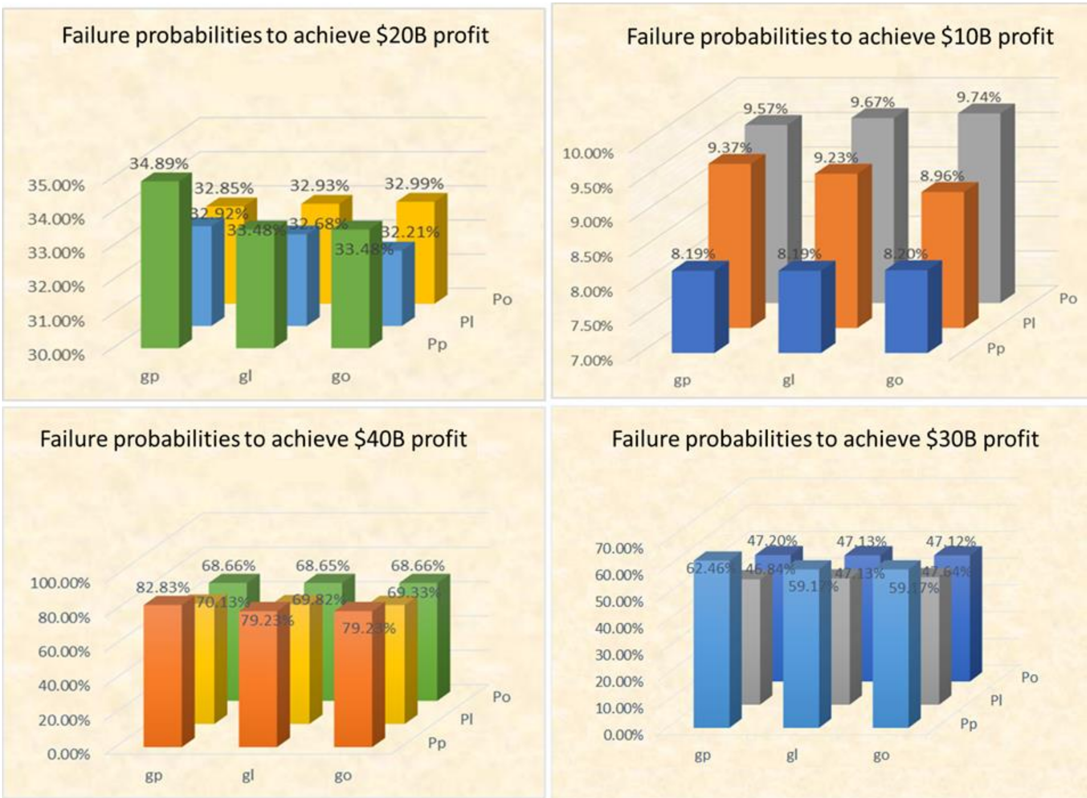

Fig. 1 . Diagram of the failure probability of nine pits according to different managerial objectives (Pp, Pl, and Po indicate pessimistic, logical, and optimistic prices, respectively, and gp, gl, and go indicate pessimistic, logical, and optimistic grades, respectively).

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