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Research article

Comparative Numerical Analysis of Indirect Tensile Strength Assessment Methods in Rock Engineering

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Abstract

This study presents an in-depth comparative numerical analysis of three distinct methods employed to evaluate the indirect tensile strength of rock materials: the Brazilian Tensile Test (BT), the Three-Point Bending Test (TPBT), and the Four-Point Bending Test (FPBT). Utilizing advanced simulation capabilities provided by the three-dimensional Particle Flow Code (PFC3D) software, the tensile behavior of rock samples was modeled and assessed under the unique loading conditions associated with each testing approach. The BT method, despite its widespread use and simplicity, revealed several limitations that could affect the reliability of its results. Key issues identified include significant stress concentration around the loading points and a non-homogeneous distribution of stress across the sample, which can introduce variability in the tensile strength measurements. In contrast, both the TPBT and FPBT methods demonstrated advantages in terms of loading control and stress distribution. The TPBT provided a more regulated loading condition compared to the BT, yet the FPBT method stood out for offering the most uniform stress distribution across the sample. The comparative analysis revealed notable discrepancies in the tensile strength values obtained from each method. Specifically, tensile strength values derived from the TPBT and FPBT were considerably different from those obtained using the BT method, with the FPBT consistently yielding the highest tensile strength measurements. These differences underscore the critical role that test method selection plays in accurately characterizing the tensile strength of rock. Overall, the study emphasizes the strengths and limitations of each testing approach, providing insights into the factors that influence tensile strength measurement outcomes. It also highlights the necessity for careful selection of the testing technique based on the specific requirements of rock mechanics analysis, particularly when precision and reliability are paramount. The findings of this research contribute to the ongoing development of more accurate and effective methods for evaluating the tensile strength of rock materials in various engineering and geological applications.

1. INTRODUCTION

Measuring and determining the properties of rocks has long presented a significant challenge for geological engineers [1]. The prevailing approach to assessing these properties involves conducting laboratory experiments and extrapolating the results to infer the in situ properties of the rock [2]. Consequently, in geological engineering, the inevitable size disparity between laboratory samples and their industrial application exists. Understanding the behavior of rocks under tensile loading and ascertaining their tensile strength is paramount for various aspects such as load-bearing capacity, deformation, fracture, crushing, and the stability of underground spaces' roofs and walls, as well as tunnel excavation, blasting, and particularly the stability of rock roofs, especially in tension zones. Therefore, the genesis of most fractures and collapses in mines, tunnels, caves, and other engineering structures can be attributed to the development of tensile stresses within them. This underscores the importance of comprehending the mechanisms of tensile fracture and devising strategies for their analysis and mitigation [3].

Various methods have been devised to measure the tensile strength of rocks, generally categorized into direct tensile and indirect tensile tests. The preferred approach is the direct tensile test, also known as the uniaxial tensile test, where the rock is directly pulled. However, this method is less favored due to the need for specialized tools and difficulties in sample preparation. The procedure for conducting this test resembles that of the uniaxial compressive strength (UCS) test, except that tensile force is applied to the sample instead of compressive force [4,5]. Over recent decades, significant research has focused on examining the compressive and tensile behavior of rock. Van Vliet, Van Mier [6] conducted direct tensile strength tests on sandrock and concrete samples ranging from 50 to 1600 mm in diameter, both in dry and saturated states. Their findings indicated that, except for 50 mm diameter concrete samples, tensile strength decreased with increasing sample diameter, although no significant trend was observed in sandrock samples. Jinmin [7] analyzed rock tensile strength using TBPT and FPBT methods, revealing uncertainties in the results obtained from bending tests. Es-Saheb et al. [8] investigated the impact of rock sample size on tensile strength through BTs and numerical analysis. Their research revealed a decreasing trend in tensile strength for samples with diameters exceeding 75 mm. Yang et al. [9] employed uniaxial tensile testing on pre-cracked rock samples to explore crack growth mechanisms using $FRANC^{3D}$ numerical simulation. Their experimental results highlighted the significant effects of pre-existing crack geometric characteristics on sample strength and failure modes. The study also examined three-dimensional crack growth patterns and rates through numerical simulations of single and double parallel cracks, demonstrating good correspondence with experimental phenomena. Allena, Cluzel [10] discussed cracking and tensile strength in cancellous bone samples ranging from 4 to 10 mm in diameter. Their research revealed a significant decrease in tensile strength with increasing sample size. Sabih et al. [11] explored the impact of sample diameter on Brazilian tensile strength using ABAQUS numerical software. Their findings revealed that the tensile strength decreases initially with an increase in sample diameter up to a certain threshold, beyond which it begins to increase. Li [12] investigated the tensile strength of Malone alluvial rocks with diameters ranging from 40 to 80 mm. The study concluded that varying the sample diameter had no discernible effect on the tensile strength of the mentioned rock. Zhou et al. [13] conducted a comprehensive study on the mechanical strength and fracture behavior of Alashan granite using both experimental laboratory tests and numerical simulations. The simulations were carried out using the grain-based approach within the two-dimensional Particle Flow Code (PFC^{2D}). This method allowed the researchers to investigate the behavior of Alashan granite under various loading conditions at a microstructural level. In their study, the microparameters for the simulation of Alashan granite were carefully calibrated to match the actual laboratory resistance values and stress-strain curves obtained from physical tests. This calibration process ensured that the numerical model closely replicated the mechanical properties observed in real samples of Alashan granite. The results indicated that it is feasible to accurately reproduce the UCS and Uniaxial Tensile Strength (UTS) of Alashan granite using the grain-based approach in PFC^{2D}. Moreover, the study revealed a positive correlation between the average mineral size within the granite and its mechanical properties, specifically UCS and UTS. This finding suggests that larger mineral grains contribute to higher strength values, providing important insights into the material behavior of granitic rocks under stress. Khosravi et al. [14] examined the influence of the length-to-diameter ratio (ranging from 0.2 to 1.5) of Brazilian discs made of gabbro, microgabbro, and basalt on fracture mechanism and surface roughness. They observed that increasing the length-to-diameter ratio led to a decrease in surface roughness in gabbro and microgabbro samples, while it exhibited a slight increase in basalt samples. Liao et al. [15] conducted a series of finite element-based three-dimensional (3D) numerical simulations to investigate the variations in tensile strength of rocks using three different test methods: the BT, the DTT, and the TPBT. These methods are commonly used in rock mechanics to assess the tensile strength, which is a critical parameter in understanding the failure behavior of rocks under tensile stress. The numerical simulations were meticulously designed to replicate the conditions of each testing method, allowing for a detailed comparison of the tensile strengths obtained from each approach. The results of the simulations revealed significant variations in the measured tensile strength depending on the test method employed. Notably, the tensile strength derived from the Three-Point Bending Test (TPBT) was found to be considerably higher than the tensile strengths obtained from the DTT and the BT. Efe et al. [16] utilized dumbbell-shaped samples to explore the impact of sample dimensions on the flexural strength characteristics of microcrystalline marble and determine DTS. Furthermore, they evaluated the indirect tensile strength of marble using BT, TPBT, and FPBT methods following EN and ASTM standards. Their study also analyzed stress distribution and intensity on the samples using ANSYS software. Golshani, Ramezanzad [17] conducted a study using the Particle Flow Code in three dimensions (PFC^{3D}) to numerically calculate the tensile strength of granite stones. The research focused on accurately modeling the mechanical behavior of granite through numerical simulations, specifically targeting Inada granite, which is sourced from a quarry in Kasama, Ibaraki, Japan. The study began by simulating uniaxial compression tests to determine the tensile strength of Inada granite. Following this, the researchers simulated the Brazilian Test conditions, which indirectly measure tensile strength by applying compressive loads along the diameter of a cylindrical rock specimen. By comparing the tensile strength results from both the uniaxial compression simulations and the Brazilian Test simulations, the researchers aimed to validate their numerical approach. The findings demonstrated that the tensile strengths numerically calculated through PFC^{3D} were in good agreement with the experimental results obtained from uniaxial tensile tests performed on actual Inada granite samples. This validation underscored the reliability of the PFC^{3D} simulations in predicting rock tensile strength and highlighted the utility of numerical methods in supplementing experimental testing. In a related study, Asadi et al. [18] examined the combined effects of loading speed and sample size on the tensile strength of rock samples with and without pre-existing cracks. They utilized both physical tests and numerical simulations using the CA3 computer program to explore these factors. Their research revealed a pronounced sensitivity of sheared rock samples to loading rate, with a critical stress rate identified beyond which the sample size no longer influenced the tensile strength. Additionally, the study observed that larger samples exhibited higher tensile strengths when subjected to loading rates exceeding this critical limit. Liu et al. [19] did a study on the development of a three-dimensional discrete element model using contact models with planar connection and smooth connection to investigate the effect of anisotropy on the tensile behavior of slate, a transversely isotropic rock and to investigate the fracture pattern, microcracks and stress distribution under the Brazilian test. They provided both macro and micro scales. Xue et al. [20] investigated the stability of artificial filling roofs made of cement tailings in underground metal mines using gold tailings and fiber-reinforced propylene fibers. Direct tensile strength and TPBT methods were conducted on the samples in the laboratory. The results indicated a significant increase in tensile and bending strength of samples reinforced with fibers. Pérez-Rey et al. [21] examined the mechanisms of tensile failure in granite rock samples across various scales using different test methodologies. They investigated granite rock samples ranging from 30 mm to 84 mm in diameter and observed a continuous increase in direct tensile strength (DTS) and rock toughness with larger sample sizes. However, no distinct trend was observed for BT. Zhang et al. [22] conducted a study using the PFC^{2D} software to investigate the discrepancies between tensile strength measurements obtained from the BT and DTS. The study aimed to understand the factors contributing to the differences in results between these commonly used rock tensile strength testing methods. Through their simulations, Zhang et al. identified that the disparity between the tensile strengths measured by BT and DTS is significantly influenced by the ratio of the rock's UCS to its DTS. Their results demonstrated a strong negative correlation between the DTS/BT ratio and the UCS/DTS ratio, indicating that as the UCS/DTS ratio increases, the discrepancy between the tensile strengths measured by BT and DTS becomes more pronounced. This relationship was consistent across various configurations of loading plates used in the BT, such as flat plates, curved jaws, and loading platforms. The study also explored the complex processes of crack initiation and propagation that occur during the BT and how these processes affect the relative relationship between BT and DTS measurements. Zhang et al. observed that both the UCS/DTS ratio of the rock and the choice of loading plate configuration significantly impact the crack initiation and propagation behavior during testing. These findings highlight the critical role of selecting the appropriate loading plate based on the UCS/DTS ratio of the rock to minimize discrepancies between the BT and DTS results. Based on their findings, Zhang et al. recommended specific ranges of UCS/DTS ratios for different BT configurations to achieve more accurate tensile strength measurements. For tests conducted with flat plates, they suggested maintaining a UCS/DTS ratio between 10 and 15. For configurations using curved jaws, a ratio of 8 to 10 was recommended, while for loading platforms, the ideal range was identified as 5 to 8. These guidelines aim to optimize test conditions, ensuring that the tensile strength values obtained from BT align more closely with those from DTS. Zhang et al. [23] conducted TPBT on rectangular pre-cracked concrete beams to investigate the propagation process of localized cracks in concrete. By monitoring the initiation and propagation of local cracks on the sample surfaces, they determined the fracture toughness of the concrete samples with local cracks and analyzed the crack propagation process in both the thickness and height directions. The experimental results revealed that under TPBT, local cracks consistently propagated first on the lower surface of the sample, forming a crack in the thickness direction. Subsequently, the crack in the sample began to propagate in the height direction until complete failure. Additionally, the initial fracture toughness obtained from bottom-cracked samples closely matched that of locally cracked samples.

Given the significant time, financial, and equipment investments required for laboratory testing, there has been a notable shift towards the utilization of numerical software in analyzing crack growth processes in rock samples. This shift has been spurred by advancements in science and technology, which have rendered traditional methods increasingly outdated. Unlike laboratory experiments, numerical simulations offer the advantage of overcoming various challenges that are difficult to address in experimental settings. To address this transition, the present study employs three-dimensional numerical investigations utilizing the PFC^{3D}-based code. The principal aim is to scrutinize and elucidate the disparities in rock tensile strength across distinct examinations, numerically, specifically the BT, TPBT, and FPBT. Subsequently, the comparative tensile strength values obtained from these three testing methodologies are evaluated. Moreover, parametric examinations and stress analyses are conducted to unveil the underlying physical mechanisms governing the numerical test outcomes, with a particular emphasis on discerning disparities between the various methods employed. The main objectives of this study are to evaluate the differences and comparative magnitudes of tensile strength obtained from various testing methods and to understand the physical mechanisms underlying these variations.

2. An OVERVIEW OF METHODS FOR THE INDIRECT DETERMINATION OF ROCK TENSILE STRENGTH

Despite numerous efforts to conduct direct tension tests accurately, this method remains technically challenging and costly. Consequently, there is a growing preference for indirect tests to determine rock tensile strength. Various methods have been developed for this purpose, all based on the principle that applying a compressive force in one direction generates a tensile force in the direction perpendicular to it. Among the indirect methods, the Brazilian method stands out as one of the most commonly used laboratory techniques for determining rock tensile strength. However, other methods are also employed, including TPBT and FPBT on cubic and cylindrical samples, as well as cylindrical and spherical diametric pressure tests, ring tests, and more. These indirect methods offer viable alternatives to direct tension tests, providing valuable insights into rock tensile strength without the technical complexities and high costs associated with direct testing.

2.1. Brazilian Tensile Test (BT)

Among the methods of indirect measurement of tensile strength, the BT stands out as a widely utilized approach for assessing the tensile strength of rocks, particularly brittle materials like concrete and rock. Originating in 1953, this method has gained prominence due to its applicability and reliability. The test involves applying diagonal pressure to cylindrical rock samples, causing tensile stress to propagate perpendicular to the loading axis. When this stress surpasses the rock's tensile strength, the sample fractures. Most rocks break under tensile stress in biaxial stress fields, making this test invaluable for indirectly measuring the uniaxial tensile strength of rock samples [24,25]. The BT is standardized by the International Society of Rock Mechanics (ISRM) and ASTM. According to ISRM guidelines, the test involves applying compressive force along the axial plane of the sample, causing it to break under induced tensile stress perpendicular to this plane. The loading force is transferred via two curved jaws in the ISRM method, while ASTM standards may utilize separate flat or curved loading plates placed directly on cylindrical specimens [24]. Initially, cylindrical samples with a diameter-to-thickness ratio of 2 are prepared and thoroughly washed. The side surfaces must be free of marks or imperfections, with dimensions less than 0.025 mm. The upper and lower surfaces should be flat, smooth, and have a maximum angle of 0.25 degrees between them. As per ISRM standards, the maximum sample diameter is 54 mm, with the radius equal to the sample thickness. To conduct the test, the sample's water content is measured, and its side surfaces are coated before placement between the curved jacks of the testing device. Loading is applied diagonally to the specimen at a constant rate, typically 200 newtons per second, according to ISRM standards. Samples typically fracture within 15-30 seconds of loading. The number of samples recommended for testing is around 10, with readings from the highest and lowest fractures included in calculations. Figure 1 illustrates the typical failure mode of rock samples in the BT [24].

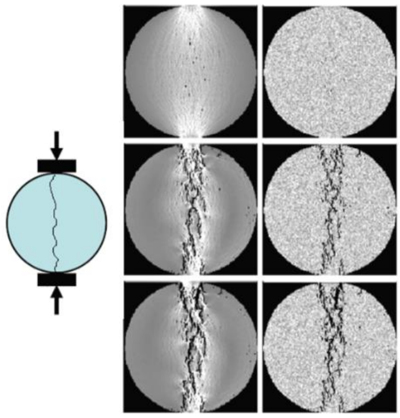

Fig. 1. The typical failure mode of rock samples in the BT.

2.2. Three-point bending test (TPBT)

In the TPBT, a sample undergoes compression for bending, leading to the development of tensile, compressive, and shear stresses within it. When only bending is applied to a portion of the sample, tensile stress occurs solely on the convex side while compressive stress occurs solely on the concave side. The highest tensile stress at the sample's breaking point is considered its tensile strength, particularly useful in assessing the tensile strength of rock formations in tunnels and mine roofs. The TPBT is a mechanical test that evaluates the bending modulus of elasticity (E_f), bending strain (ε_f), and bending stress (σ_f). Standard devices such as the universal tensile testing device are used for this test, arranged in various configurations like TPBT, and FPBT. While TPBT offers the advantage of easy sample preparation, its results are sensitive to sample geometry and test speed. Tests are conducted according to standards such as TS EN 12372 (flexural strength under concentrated load) and ASTM C99. The TS EN 12372 standard specifies criteria such as thickness (h), total length (L), width (b), and distance between holding rollers (l).

The numerical modeling of the TPBT, illustrated in Figure 2, encompasses various parameters such as beam length (lb), depth (d), width (b), and beam span (l). Compression forces are applied along the top centerline of the rock beam, with support provided near the ends at the bottom. Initial failure usually occurs at the bottom center of the beam, allowing the tensile strength of the rock to be determined using Equation (1), where σtt denotes the three-point bending strength. For samples with a circular cross-section, tensile strength can be calculated using Equation (2), where σt represents tensile strength, F is the applied force exerted by the moving arm, L is the distance between the supporting bases, and R is the radius of the beam.

$$\sigma_{tt} = \frac{3P_c l}{2bd^2} \tag{1}$$

$$\sigma_t = \frac{FL}{\pi R^3} \tag{2}$$

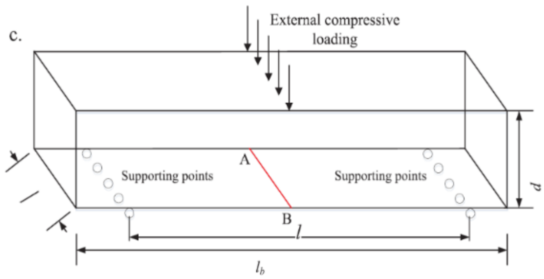

Fig. 2. The numerical model of TPBT [26].

Additionally, the fracture toughness of a sample can be determined using TPBT. As shown in Figure 3, the stress intensity coefficient at the location of a crack can be expressed using Equation (3), where P represents the applied load, B is the thickness of the sample, a is the crack length, and W is the sample width. In TPBT, the desired crack is created through cyclic loading and sample fatigue at the desired location. The crack length is measured, and then the sample is uniformly loaded. The force at which the crack starts to grow is used to determine the resistance against material failure using Equation (4), where Y is calculated using Equation (5).

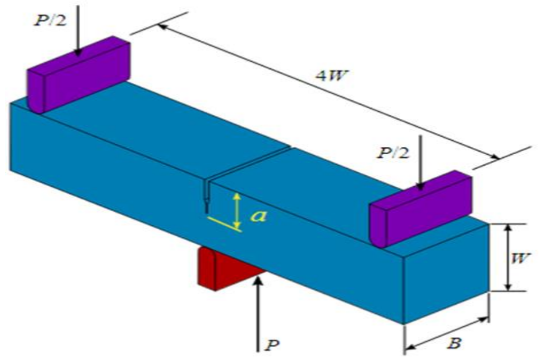

Fig. 3. A bending cracked specimen (single edge) used for fracture resistance testing [26].

$$k_1 = \frac{4P}{B}\sqrt{\frac{\pi}{W}}\left[\begin{array}{l}\left(1.6\frac{a}{W}\right)^{\frac{1}{2}} - \left(2.6\frac{a}{W}\right)^{\frac{3}{2}} + \left(\frac{a}{W}\right)^{\frac{5}{2}} \\ -\left(21.2\frac{a}{W}\right)^{\frac{7}{2}} + \left(21.8\frac{a}{W}\right)^{\frac{9}{2}}\end{array}\right] \quad (3)$$

$$k_I = \frac{6P}{BW}a^{\frac{1}{2}}Y \quad (4)$$

where Y is equal to:

$$Y = \frac{1.99 - \frac{a}{W}((1-\frac{a}{W})(2.15 - 3.93\frac{a}{W} + 2.7\frac{a}{w}))}{(1+2\frac{a}{W})(1-\frac{a}{W})^{\frac{3}{2}}} \quad (5)$$

2.3. Four-point bending test (FPBT)

In FPBT, which evaluates the bending resistance of materials, standard universal tensile testing devices are utilized, similar to other bending tests. However, unlike the three-point bending test, the FPBT employs two rollers to apply force, ensuring uniform loading and preventing stress concentration. This configuration divides the sample into three equal parts, with the loading points on the top of the sample placed at equal distances. The FPBT follows standards such as TS EN 13161 (flexural strength under constant moment) and ASTM 880-98, maintaining similar sample dimensions and loading parameters as the three-point bending test (TS EN 12372 standard).

As depicted in Figure 4, the numerical model of the FPBT presents a rectangular cube sample with specified parameters including lb, d, b, and l. Initial failure typically manifests at the bottom center of the rock beam, facilitating the calculation of the corresponding rock's tensile strength using Equation (6), wherein σ_t signifies the three-point bending strength. In this equation, P_c represents the peak compressive load, l denotes the beam span, while b and d refer to the width and depth of the rock beam, respectively.

$$\sigma_{tt} = \frac{3P_c l}{4bd^2} \quad (6)$$

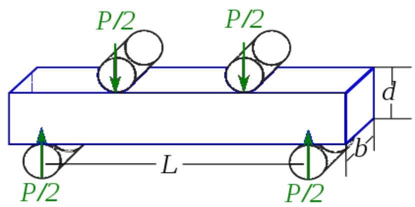

Fig. 4. The numerical model of FPBT [27].

For samples with a circular cross-section, the tensile strength can be calculated using Equation (7), where σ_t represents the tensile strength (Pa), F is the force applied by the moving arm (N), L is the distance between the two supports (points) (m), and R is the radius of the sample beam (m).

$$\sigma_t = \frac{8FL}{\pi R^3} \quad (7)$$

3. EXPLORING ROCK TENSILE STRENGTH VARIATION THROUGH DIVERSE TESTING TECHNIQUES

Pstudy Flow Code (PFC) models are comprised of an assembly of rigid pstudys with diverse sizes, engaging in interactions through contacts to replicate the behavior of granular and solid materials. These models facilitate the simulation of individual motion and interaction among numerous rigid pstudys, where interactions are regulated by internal forces and moments. Pstudy shapes encompass various geometries such as 2D disks or 3D spheres, along with interconnected disks forming collections in 2D or 3D spheres, and convex polygons in 2D or polyhedra in 3D. Contact mechanics within PFC models adhere to fundamental principles governing pstudy interactions, ensuring accurate updates of internal forces and moments. The versatility of PFC allows for customization and application across a diverse spectrum of numerical investigations where discrete system behavior is of paramount importance. Since its establishment in 1994, PFC has risen as a prominent DEM tool in geological research, covering a broad spectrum from fundamental investigations into fine-scale soil and rock behavior to a plethora of large-scale applications. These applications include hydraulic fracturing, interactions between soil and tools, fracture mechanics of brittle rocks, analysis of slope stability, drilling operations, rock cutting, pavement design, material handling, dynamics of bulk material flow, and simulations of cave mining. Numerical methods, particularly the Discrete Element Numerical Method, are favored for their adaptability in tackling complex engineering challenges. PFC^{3D}, among the suite of software platforms grounded in the Discrete Element Method, stands out as a robust tool for addressing discontinuous environmental conditions prevalent in geotechnical engineering. Notably, PFC^{3D} offers the capability to model a discrete fracture network (DFN) and derive material behavior characteristics based on laboratory-scale macro properties and calibration procedures. These advanced capabilities enable the creation of highly realistic models that closely mirror real-world conditions, resulting in more precise and reliable outcomes.

This study focuses on investigating discrepancies in rock tensile strength obtained from three distinct testing methodologies: BT, TPBT, and FPBT, along with an examination of their respective underlying physical mechanisms.

3.1. Modeling and Analysis of BT

To perform the above test, disk samples with a diameter of 54 mm and a thickness of 27 mm were used along with the selected parameters for the loading plate radius and loading rate. The material properties, including Poisson's ratio and modulus of elasticity, are also specified. The boundary conditions for the model are set such that only vertical displacement is allowed at the floor.

Figure 5 depicts the sample after loading, showing the occurrence of a fracture perpendicular to the direction of force application, resulting in a tensile crack. Biaxial loading is observed at the beginning and end of the sample due to the curvature of the loading plane.

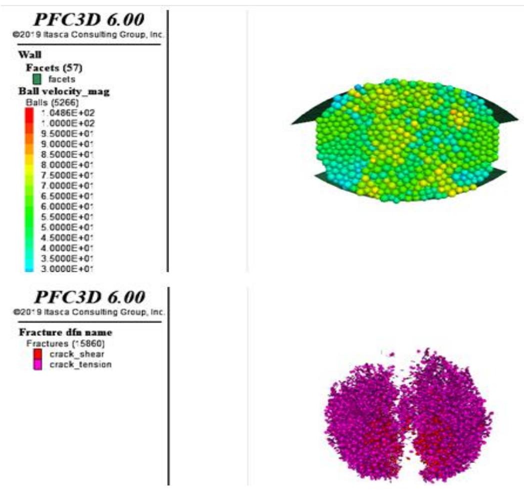

Fig. 5. Brazilian Disc After Load Application in BT.

Further analysis in Figure 6 reveals that stresses in the (xx) direction induce tensile cracks perpendicular to the loading plane. Conversely, Figure 7 demonstrates minimal stress and displacement in the (yy) direction, indicating a negligible contribution to the failure behavior.

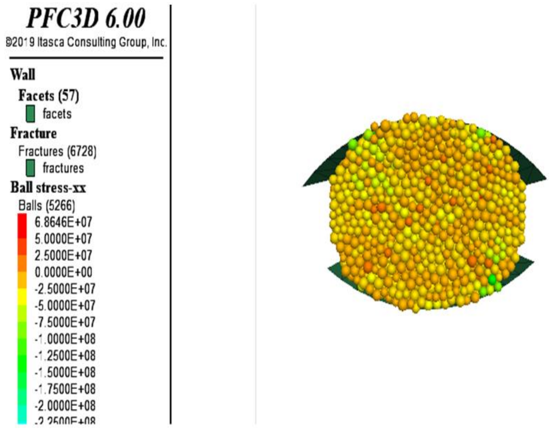

Fig. 6. Brazilian disc after applying load in (xx) direction.

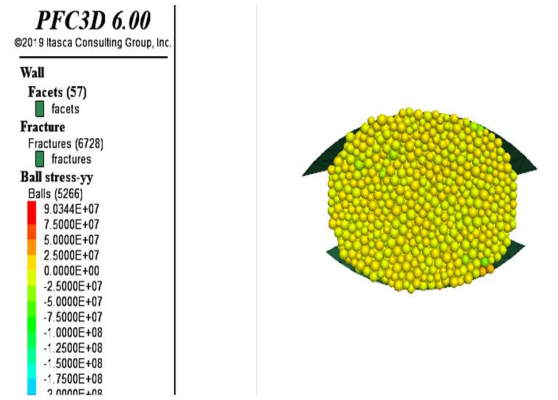

Fig. 7. Brazilian disk after applying load in (yy) direction.

Examining stresses in the (zz) direction, Figure 8 shows maximum stresses aligning with the applied load direction, leading to failure perpendicular to this direction. Figure 9 illustrates displacement patterns, with tensile displacements occurring perpendicular to the loading direction, indicative of sample fracture.

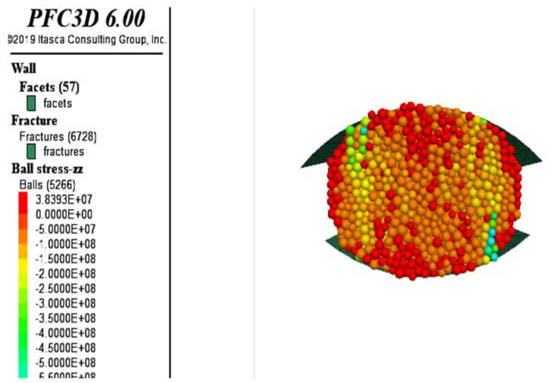

Fig. 8. Brazilian disk after applying the load in the (zz) direction.

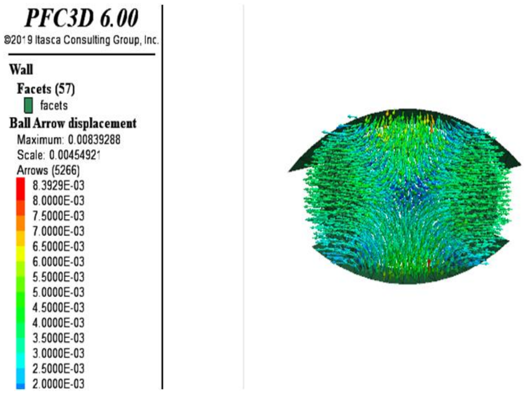

Fig. 9. Displacement of the Brazilian disc after loading.

The model calculates the displacement of contact nodes with the upper and lower jaws, determining permanent strain. Additionally, vertical stress at the contact zones and stress-strain curves, as shown in Figure 10, are analyzed.

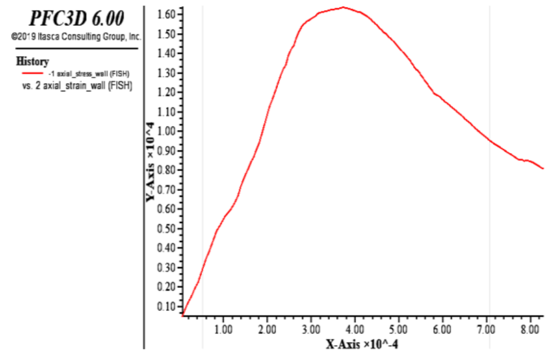

Fig. 10. Force-Displacement Diagram for BT.

According to the force-displacement diagram in Figure 10, the sample exhibits linear elastic behavior until reaching a force of 13 KN, beyond which failure occurs at a maximum applied load of 16 KN. The tensile strength obtained from the BT is determined to be 6.88 MPa using Equation (8).

$$\sigma_t = \frac{2p}{\pi dt} = \frac{2\times 16\times 10^3}{3.14\times 54\times 27\times 10^{-6}} = 6.88MPa \qquad (8)$$

3.2. Modeling and Analysis of TPBT

In the modeling and analysis of TPBT, cubic samples with dimensions of 150 mm length, 25 mm width, and 50 mm thickness were used (Figure 11). The loading rate was set to 50 N/s, and consistent with the BT model, the modulus of elasticity was considered to be 18.6 GPa with a Poisson's ratio of 0.25. Boundary conditions were established such that the model floor experienced constant displacement solely in the vertical direction (z-axis).

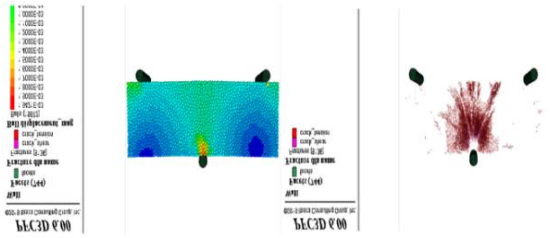

Fig. 11. Rectangular Cuboid Post-Loading.

Stresses along the (xx) direction are shown in Figure 12, indicating cracks perpendicular to the loading plane and in the (xx) direction. Notably, a transition from uniaxial to biaxial stress was observed at the center end of the sample, reducing test accuracy.

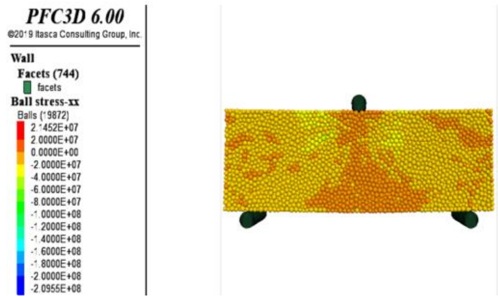

Fig. 12. Rectangular Cuboid Post-Loading in the xx Direction.

Figure 13 illustrates negligible stresses in the (yy) direction, resulting in minimal displacement and no significant failure mechanism in this direction.

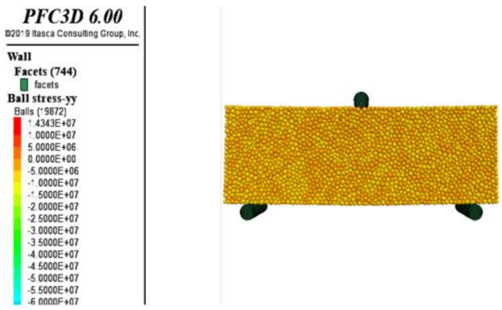

Fig. 13. Rectangular Cuboid Post-Loading in the yy Direction.

Stresses along the (zz) direction, depicted in Figure 14, show maximum tension, causing the sample to break perpendicular to this direction.

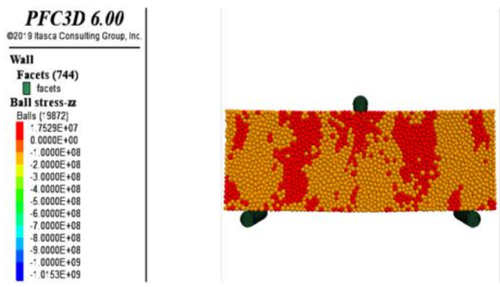

Fig. 14. Rectangular Cuboid Post-Loading in the zz Direction.

Displacement of the sample, depicted in Figure 15, initiates from the load application area, culminating in a tensile crack.

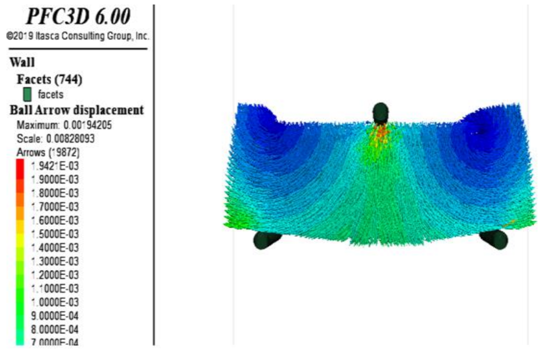

Fig. 15. Displacement of the Rectangular Cuboid Post-Loading.

The force-displacement diagram in Figure 16 indicates that the sample remains within the linear elastic region until a force of 1.8 KN, beyond which deformation occurs, ultimately resulting in failure at 4.3 KN force.

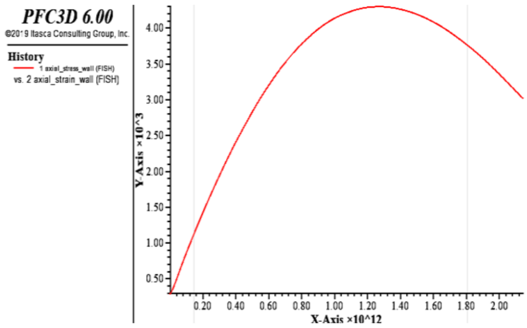

Fig. 16. Force-Displacement Diagram for TPBT.

The tensile strength obtained from the three-point bending test is calculated as 15.48 MPa, as per Equation (9).

$$\sigma_t = \frac{3FL}{2bd^2} = \frac{3\times 4.3\times 10^3 \times 150\times 10^{-3}}{2\times 25\times 50^2 \times 10^{-9}} = 15.48 MPa \quad (9)$$

3.3. Modeling and Analysis of FPBT

In this section, to perform the FPBT test, the loading rate was set at 10 N/s, with a constant modulus of elasticity of 18.6 GPa, and a Poisson's ratio of 0.25, according to BTs. The boundary conditions are configured to apply a constant vertical displacement only along the z-axis on the floor of the model.

Post-loading, as shown in Figure 17, fractures appeared perpendicular to the applied force direction, resulting in a tensile crack formation within the sample. Due to loading from two points, the sample segmented into three distinct parts.

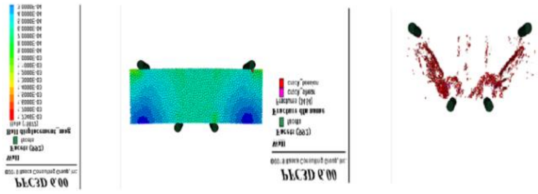

Fig. 17. Rectangular Cuboid Post-Loading.

Figure 18 depicts stress distribution along the (xx) direction, indicating crack formation perpendicular to the loading plane and specifically along the (xx) axis. Notably, the highest tensile stress occurred at the terminus of the two loading points.

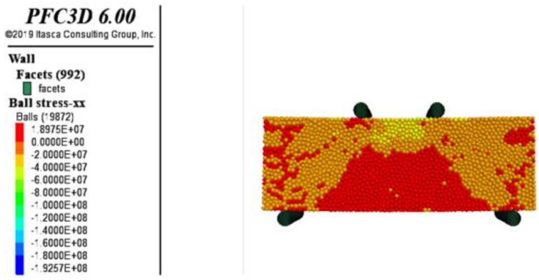

Fig. 18. Rectangular Cuboid Post-Loading in the xx Direction.

Stresses along the (yy) direction, as demonstrated in Fig. 19, were minimal, leading to negligible displacement and no significant impact on the sample's failure behavior.

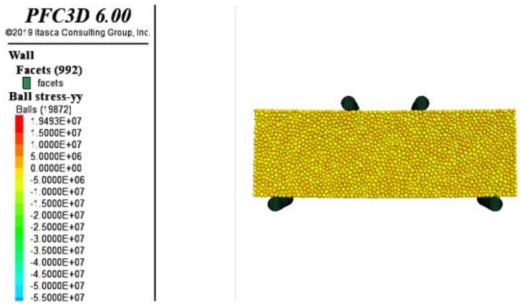

Fig. 19. Rectangular Cuboid Post-Loading in the yy Direction.

In Figure 20, maximum stresses along the (zz) direction were observed, indicating the direction of the applied load.

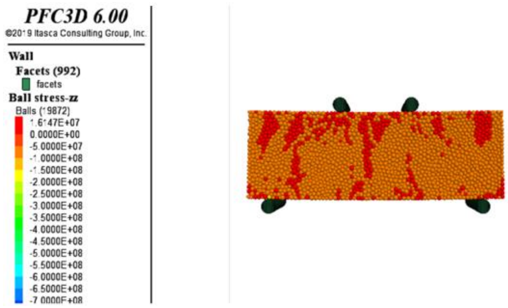

Fig. 20. Rectangular Cuboid Post-Loading in the zz Direction.

Figure 21 illustrates the displacement pattern within the sample, originating from the load application areas and culminating in the formation of a tensile crack.

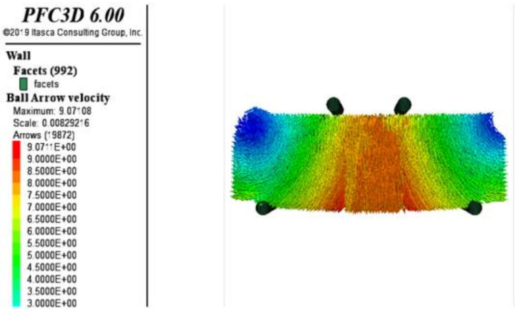

Fig. 21. Displacement of the Rectangular Cuboid Post-Loading.

The force-displacement diagram depicted in Figure 22 exhibits a linear elastic region until the force reaches 5.6 KN, beyond which deformation initiates, leading to ultimate failure at 7.4 KN.

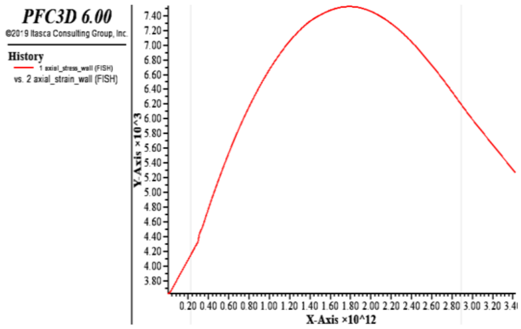

Fig. 22. Force-Displacement Diagram for FPBT.

The tensile strength obtained from the BT, as per Equation (10), is calculated to be 13.2 MPa.

$$\sigma_t = \frac{3FL}{2bd^2} = \frac{3\times 7.4\times 10^3\times 150\times 10^{-3}}{4\times 25\times 50^2\times 10^{-9}} = 13.2MPa \qquad (10)$$

4. DISSCUSION

Rock mechanics and engineering rely heavily on accurate assessments of tensile strength to understand material behavior and ensure structural stability. The BT, TPBT, and FPBT are commonly used methods for evaluating the tensile strength of rocks. In this comparative analysis, we will delve into the principles, procedures, and applications of each testing method to highlight their relative merits and drawbacks.

Brazilian Tensile Test (BT):

The BT, also known as the indirect tensile strength test, is widely used for assessing the tensile strength of rocks. In this test, a cylindrical rock specimen is subjected to diametrical compression, resulting in tensile failure along the diametrical plane perpendicular to the applied load. The test setup involves placing the rock sample between two parallel platens of a testing machine, with a compressive force applied diametrically until failure occurs. The tensile strength of the rock is calculated based on the maximum load sustained before failure and the dimensions of the specimen.

Advantages of the BT:

Simple and straightforward test setup, requiring minimal specimen preparation.

- Provides a direct measurement of tensile strength, which is crucial for assessing rock stability.
- Widely accepted and standardized testing method in the field of rock mechanics.
- Suitable for a wide range of rock types and sizes, making it versatile for various applications.

Limitations of the BT:

- Assumes homogeneous material properties across the specimen, which may not always be accurate for natural rock formations.
- Vulnerable to misalignment and eccentric loading, leading to inaccurate results.
- Limited to relatively small sample sizes, restricting its applicability for large-scale projects.
- Does not account for the influence of confining pressure or complex stress states on tensile strength.

Three-Point Bending Test (TPBT):

The TPBT is another commonly used method for evaluating the tensile strength of rocks. In this test, a prismatic rock specimen is supported by two parallel platens, with a third point load applied at the center of the specimen. As the load is gradually increased, tensile stresses develop on the underside of the specimen, leading to crack initiation and propagation. The tensile strength of the rock is determined based on the applied load and the dimensions of the specimen.

Advantages of the TPBT:

- Allows for the assessment of tensile strength under controlled loading conditions, facilitating accurate measurements.
- Can accommodate larger sample sizes compared to the BT, making it suitable

for testing rocks with varying geometries.
- Provides insights into crack initiation and propagation behavior, aiding in fracture mechanics studies.
- Offers flexibility in test configurations, allowing researchers to customize loading conditions based on specific requirements.

Limitations of the TPBT:

- Requires precise alignment of the loading and support points to avoid eccentric loading effects.
- Susceptible to edge effects and stress concentrations near the loading points, potentially influencing test results.
- May underestimate tensile strength due to the presence of compressive stresses on the upper surface of the specimen.
- Limited applicability for rocks with non-prismatic shapes or irregular geometries.

Four-Point Bending Test (FPBT):

The FPBT is a modified version of the TBPT, offering improved control over stress distribution and crack propagation. In this test, the specimen is supported by two outer loading points and two inner support points, creating a more uniform stress distribution along the length of the specimen. As the load is applied, tensile stresses develop on the underside of the specimen, leading to crack formation and failure.

Advantages of the FPBT:

- Provides more uniform stress distribution compared to the TBPT, reducing the influence of stress concentrations.
- Allows for the testing of larger and non-prismatic specimens, expanding its applicability to a wider range of rock types and geometries.
- Offers better control over crack initiation and propagation behavior, leading to more reliable tensile strength measurements.
- Minimizes edge effects and eccentric loading, resulting in more accurate and consistent test results.

Limitations of the FPBT:

- Requires more complex test setup and instrumentation compared to the TBPT, increasing experimental complexity.
- May still be susceptible to misalignment and eccentric loading if not carefully executed.
- Limited availability of standardized testing procedures and guidelines, requiring careful experimental design and validation.

Comparative Analysis and Conclusion

In summary, each of the three testing methods offers unique advantages and limitations in assessing the tensile strength of rocks. The BT provides a direct measurement of tensile strength and is widely accepted in the field, but it may not accurately represent the tensile behavior of all rock types. The TBPT allows for controlled loading conditions and provides insights into crack initiation and propagation, but it may underestimate tensile strength due to compressive stresses. The FPBT offers improved stress distribution and crack control, making it suitable for testing larger and non-prismatic specimens, but it requires a more complex setup and instrumentation.

Ultimately, the choice of testing method should be based on project requirements, specimen characteristics, and research objectives. Researchers and engineers should carefully consider the advantages and limitations of each method to ensure accurate and reliable assessment of rock tensile strength in various applications.

5. CONCLUSION

In comparing the numerical modeling of the BT, TPBT, and FPBT, several critical factors emerge that influence the accuracy, reliability, and applicability of these methods in rock engineering analysis.

Starting with the BT, numerical simulations involve modeling the cylindrical rock specimen subjected to diametrical compression. The numerical model accurately represents the loading conditions, material properties, and boundary conditions, allowing for a detailed analysis of crack initiation, propagation, and failure mechanisms. However, challenges arise in accurately capturing the complex stress distribution and strain localization near the loading points, which can affect the interpretation of tensile strength values.

Moving to the TPBT, numerical modeling focuses on simulating the application of a point load to the center of a prismatic rock specimen. The numerical model enables precise control over loading parameters, specimen geometry, and material behavior, facilitating a comprehensive investigation of stress-strain responses and failure modes. Nevertheless, challenges persist in modeling the contact interactions between the loading point and the specimen surface, as well as in accurately predicting crack initiation and propagation under bending conditions.

In the case of the FPBT, numerical simulations involve modeling the specimen supported at two inner points and loaded at two outer points, aiming to achieve more uniform stress distribution and crack control compared to the TPBT. The numerical model allows for detailed analysis of stress concentrations, crack development, and failure mechanisms, providing insights into the effectiveness of the FPBT in assessing tensile strength. However, challenges arise in accurately capturing the interaction between the loading and support points, as well as in accounting for geometric nonlinearity and material heterogeneity in the numerical model.

Overall, numerical modeling offers a powerful tool for comparing the BT, TPBT, and FPBT in rock engineering applications. By accurately simulating the loading conditions, material behavior, and failure mechanisms, numerical simulations provide valuable insights into the strengths and limitations of each testing method. However, challenges remain in accurately representing the complex interactions and phenomena inherent in rock mechanics, underscoring the need for further research and development to enhance the accuracy and reliability of numerical models in rock engineering analysis.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This study does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

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