Discontinued Rock Slope Analysis through a New TFS-KGM Analytical Method

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Abstract

Slope stability analysis of jointed rocks has been the focus of many studies. The presence of joints and discontinuities in rock environments intensifies instability along with the development of block movements. Many analytical and numerical methods have been proposed and applied to analyze the stability of jointed rock slopes. Computation complexity, incapability of presenting a reliable safety factor to be used for developing a proper design operation and improper analysis speed are the known challenges of these methods. This paper has developed the well-known analytical Key Group Method (KGM) to Toppling-Free fall-Sliding Key Group (TFS_KGM) version. To this end, toppling and free fall failure are added to the existing method in order to have a better analysis of jointed rock slopes. In this method, unstable key blocks participate in creating groups which may rotate or free fall besides sliding. The new TFS_KGM computes stability conditions and final safety factors based on the most unstable sliding, rotating and free fall movements with consideration of in situ stresses. Results of using this method in jointed rock slopes and the comparison with DEM numerical, KBM and KGM analytical methods show that the method is very effective particularly when the geometrical conditions of the jointed rocks make the toppling and free fall failures potentially possible. The method demonstrates a simple computation method along with a proper analysis speed. It also provides accurate design safety factors and much more optimized critical failure areas than previous methods.

1. INTRODUCTION

Analytical methods are fast and simple to use for the analysis of rock slopes. These methods have been thoroughly discussed in many studies. Duncan [1], Hungr et al. [2,3], Chen and Chameau[4] Lam and Fredlund[5] extended Bishop, Spencer, Morgenstern's limit equilibrium methods. The Key block method (KBM) developed by Goodman is an effective method in the analysis of jointed rock slopes and underground spaces. This method assumes all blocks to be rigid in the procedure of the analysis [6,7]. Simplicity and accuracy of the method besides its appropriate computation speed in comparison with other often complicated continuum and discontinuum analyses used in the numerical methods such as finite element, finite difference, discrete element methods, etc. are the advantages of the key block method [8]. In previous studies, the implementation of key block analysis is performed either through vector analysis proposed by Warburton [7] or graphical technique developed by Goodman and Shi [6]. The main object in all these methods is to find and extend the analysis based on some specific blocks (key blocks) of the whole set of blocks in the rock mass. The specific blocks have the following characteristics; they have direct contact with excavated surface (active blocks), they are confined by discontinuities and excavation (finite blocks), they are geometrically moveable and the movement of another block depends on this specific block. A block with all these characteristics is called a key block. If an unstable key block is not supported, its movement can cause instability of other blocks.

In the first studies of key blocks, Mauldon et al. [9] discussed the formed key blocks in a tunnel in 2D. Tanon[10] generalized Goodman's and Mauldon's vector analysis of key block rotations.

Sagaseta et al. [11] proposed a general analytical solution for the required anchor force in rock slopes with toppling failure. In newer studies González-Palacio et al. [12] proposed a new geometrical technique in key blocks method. Their technique can identify and analyze pyramidal and non-pyramidal blocks in underground spaces. In this technique, none-pyramidal blocks are identified using a virtual geometrical movement in blocks. In another study, they extended their technique to tetrahedral and pentahedral key blocks for a 3D analysis [13]. Zhang et al. [14] tried to identify key blocks with more details on jointing conditions and other weak planes and also the irregular geometry of the project. Therefore, they proposed a combination of key block and finite element methods. Their proposed method can identify and analyze irregular convex and concave blocks. Greif and Valcko used key block theory for rock slope stability analysis in the foundations of medieval castles in Slovakia [15]. They calculated risk potential besides static and quasi-static analysis of the site. Wang et al. [16] combined Monte Carlo technique with key block method to analyze the stability of a tunnel blocks.

The application of key blocks bears two limitations; First, The criterion for extension of the computation is the condition of the key block. If the key block is stable the whole group is considered stable while the combination of the neighboring blocks may pose a more critical condition. Second, ignoring the effect of in situ stresses has resulted in inappropriate results and unrealistic safety factors [9]. There have been a few studies on improving the key block method with regard to these limitations. Wibowo proposed to consider secondary blocks [17]. In other studies, it is recommended that all blocks neighboring the key block should be taken into consideration for the formation of a more critically stable key group. This is the first step towards proposition of the key group method (KGM).

Key group method proposed by Yarahmadi and Verdel improves the analysis of key elements in key blocks method [8]. This method takes neighboring blocks of the key block into consideration too. It investigates the possibility of the formation of unstable groups consisting of the key block and its neighboring blocks, therefore, it removes the first limitation (considering the key block only) of key block method. Yarahmadi and Verdel extended their method to consider uncertainty in mechanical parameters and proposed probabilistic key-group method (PKGM) [18]. They also added a Sarma based analysis for key group method to consider inter-block forces [19]. Key group method was further extended to three dimensions by Noroozi et al. [20].

Ignoring in situ stresses and incapability of analysis in the presence of concave blocks are the limitations of the key group method. Also, the key group method (like key block method) only considers sliding instability which only appears in a fraction of joints orientation. Therefore, the method is not applicable in all other cases of instability (i.e. toppling and free fall failure of the unstable key group). Toppling failure has been studied by Hoffman (1972), Ashebi (1971), Suto (1974), Cundall (1974), Beiren (1974), Hammet (1974) [21]. They have tried to propose a model besides studying rotating blocks in pure toppling failure. However, the proposed models are very simple and cannot be extended for real jointed blocks.

This study proposes a comprehensive model called "Toppling-Free fall-Sliding key group method" (TFS_KGM). This method provides occurrence of toppling and free fall instabilities besides sliding. In this method, the model key blocks calculate based on triple mobility modes, sliding, toppling and free fall. Then, with key group's creation consisting of the key blocks and their neighbors, their mobility modes are detected. This process continues until the most instable groups are identified. In this case, the group is removed. Then, The calculations end if no more instable group is found. Given the role of in situ stress in the instabilities, TFS-KGM, considers the issue of in situ stresses. Moreover, there is no limitation about block concavity. The development of this method is discussed below.

2. EXTENDING TFS_KGM METHOD

Toppling free-fall sliding key group method (TFS_KGM) is a limit equilibrium method based on the key block method. The method considers all failure modes and natural joint orientations. Therefore, it can be considered as an effective method for the analysis of sliding, toppling, free-fall failures and any combination of them. The key group method considers in situ stresses based on their role on the extension of a toppling condition. However, blocks are assumed to be rigid due to low values of in situ stresses in relation to high depths and lack of block deformation in this condition. This method provides accurate safety factors in a simple and fast analysis. The proposed method is presented in a computation pack for MATHEMATICA. The computation pack extends the analysis procedure from geometrical modeling to final mechanical analysis. The method will be discussed in details.

2.1. Geometrical Modeling

Geometrical modeling in TFS_KGM includes modeling of the boundaries, and then modeling of discontinuity systems. The developed algorithm has no limitation in geometrical modeling. Therefore, it is possible to describe the topographical surfaces using geometrical points in the algorithm. Since it is necessary to consider real jointing condition besides the manual description of joint segments, simulation of discontinuity system is carried out using two methods; infinite non-sequential discontinuity system, and sequential discontinuity system. In the non-sequential infinite system, joint sets do not have priority. Hence all the discontinuities intersect themselves. While in sequential infinite system, some joint sets intersect the others due to the genesis priority. In this case, the sets that are cut are known as secondary and the cutter sets are known as primary sets.

In these two systems, four parameters of joint dip (a_m), the standard deviation of joint dip (a_d), average joint spacing (s_m), and standard deviation of joint spacing (s_d) are used for the description of a joint set. Orientation and spacing of a joint set is described based on Gaussian statistical distribution. The least horizontal and vertical position of the block model is described as the center of the joint set (c_j). A navigation path, perpendicular to the average dip of the joint set is defined and used for determination of the center of each individual joint. During a stepwise procedure, based on input parameters, random data of spacing (S_i), and dip angle (a_i) are produced according to a normal distribution. The produced random data are used to add individual joints to the geometrical model. The procedure is shown schematically in Fig. 1.

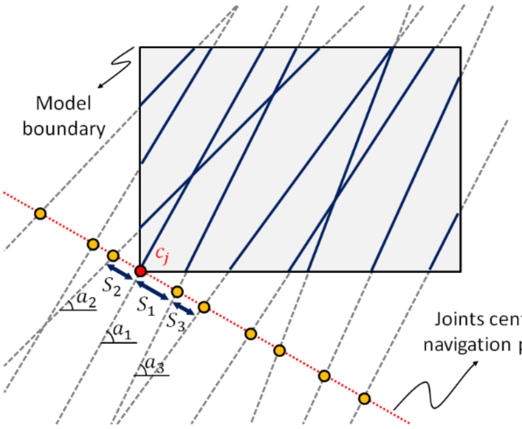


Figure 1. The simulation procedure for an infinite joint set

In the simulation of an infinite non-sequential jointed system, distributions of joints in different joint sets of the model are completely independent of each other. Therefore, the proposed procedure is exactly repeated for other joint sets. Spatially independent distribution of joint sets is not accurate enough because of the priority of the main joint sets to secondary joint sets. Hence, the geometrical modeling algorithm is designed in a way that in each step (after the distribution of the main joint), the formed clusters of the model are detected and the procedure of the secondary joint sets is followed for all blocks individually. The schematic of the produced secondary joint sets in sequential and non-sequential jointing systems is presented in Fig. 2. Furthermore, a simulation sample of the two mentioned algorithms in an environment with three joint sets is presented in Fig. 3.

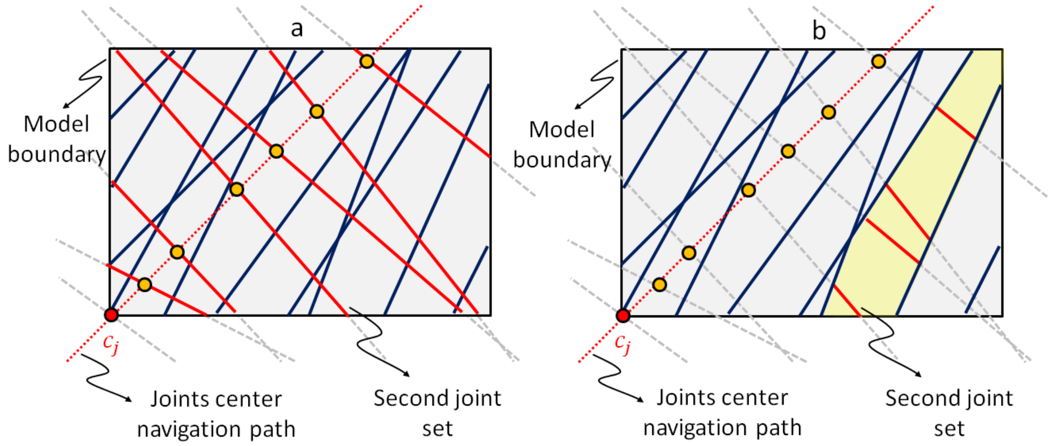


Figure 2. Simulation algorithm of the secondary joint sets, a) infinite non-sequential jointing system b) infinite sequential jointing system

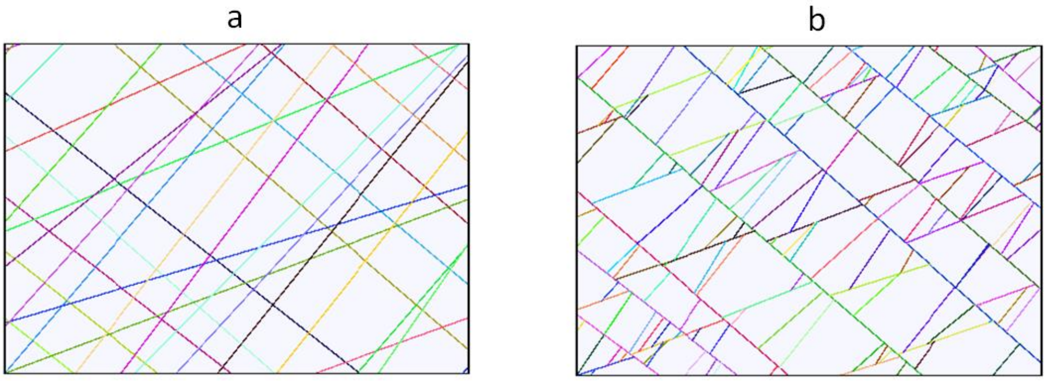


Figure 3. Statistical simulation of a hypothetical infinite a) non-sequential jointing system b) sequential jointing system.

Other geometrical modeling procedures begin after the simulation of joint segments and statistical placement of them on the understudy model. In order to extend the mechanical analysis in TFS_KGM, blocks created by the intersection of the joints and model boundaries should be detected. So as the first step, separation and segmentation of joint segments created from the intersection of joints with each other and model boundaries should be done. In order to do that, segmentation algorithm navigates through joints produced in previous stages and detects the intersections of them. Finding the intersection points, the algorithm may detect the smallest joint segments. Then during a course of stepwise procedure, a navigation begins through all vertices of the joint segments and continues to all segments. In this navigation, local coordinates $x'y'$ are described on arrival to any vertex in such a way that x'-axis is in opposite direction of the last navigation path. The local angles of the joint segments passing through the origin of the local coordinate system are calculated and the smallest angle is chosen as the next navigation path (i.e. the next edge of the block is chosen). The procedure continues until reaching the initial vertex (see Fig. 4). It should be noted that since TFS_KGM considers both convex and concave blocks, the algorithm can detect both kinds of the blocks. Fig. 5 demonstrates the procedure of formation of joint segments and detected blocks in a hypothetical block set.

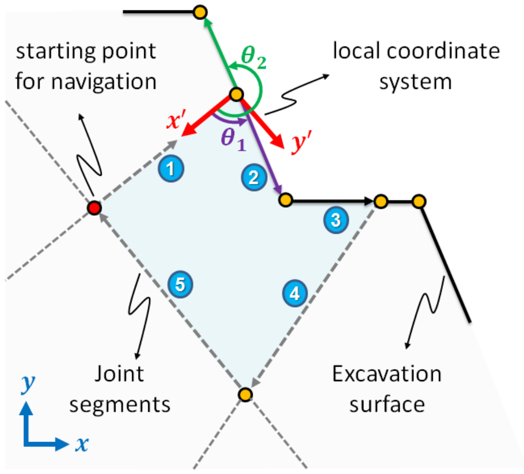


Figure 4. Schematic of a block detection procedure based on joint segments

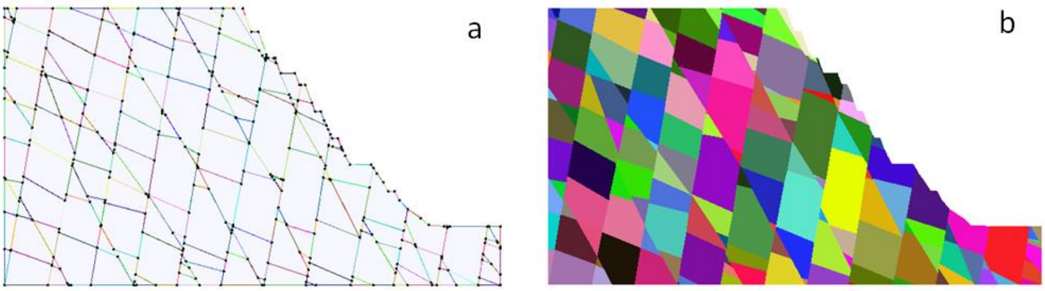


Figure 5. a) Joint segment formation procedure b) Detection of blocks in a block set

Center of mass and area of a block are of the most important geometrical properties which are used in the calculation of the weight and momentum of a block or a group. According to the recommendation of Bourke et al. [22] the area of a polygon with n vertices and coordinates (xi,yi) (i=1,n) can be calculated using Eq. (1).

$$A = 1/2\left|\sum_{i=1}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)\right| \tag{1}$$

The centroid of a polygon may be calculated once the area is determined.

$$C_x = 1/6A\sum_{i=1}^{n-1}(x_i + x_{i+1})\,(x_i y_{i+1} - x_{i+1} y_i) \tag{2}$$

$$C_y = 1/6A\sum_{i=1}^{n-1}(y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \tag{3}$$

where (C_x, C_y) are the coordinates of the polygon centroid. Considering the necessity of closing the polygon $(x_1,y_1)= (x_n,y_n)$, the above equations develop the calculation procedure in different conditions of convexity of the blocks.

2.2. Calculation of Acting Forces on Blocks

One of the objectives for the development of TFS_KGM is consideration of stresses in the analysis. Stresses are caused by overburden weight, they develop inter-block forces in block edges. Generally, in this analysis, forces acting on a block (bulk forces such as block weight) and inter-block forces are due to the presence of the initial in situ stresses and transition of the stresses through adjacent blocks. Since the rotation and instabilities conditions related to toppling failure has been added to the method, traces of inter-block forces and bulk forces are of paramount importance. Therefore, in addition to accurate calculation of these forces, their trace point (stress center) should also be calculated. So the procedure of calculation forces acting on a block consists of two parts; calculation of magnitude of forces and trace point.

In this study, the magnitude of forces acting on a block are calculated assuming all materials have similar densities, and deformations and displacements have not occurred yet (initial stresses have not relaxed). Gravity loading technique has been used for calculation [23]. It is assumed that the existence of joints in the initial in situ mass makes no disturbance in the transition of stress to lower layers. Taking this assumption, stresses are calculated based on the height of the overburden column. It should be noted that because the free faces of the problem (initial polygon) are discretized and necessity of accurate measurement of overburden height in each point, interpolation function F is used in the middle of each of two consecutive points on the free face. This function performs a linear interpolation between two consecutive points p_i and p_{i+1}. It also calculates the height of surface (h_i) continuously (Fig. 6(a)). Eqs. (4) and (5) describe stress calculation based on F function in arbitrary point x_j,y_j.

$$\sigma_y(x_j, y_j) = \rho \times g \times (F_{i,i+1}(x_j) - y_j) \tag{4}$$

$$\sigma_x(x_j, y_j) = k_0 \times \rho \times g \times (F_{i,i+1}(x_j) - y_j) \tag{5}$$

ρ is material density, g is gravity, k_0 is horizontal stress coefficient, $F_{i,i+1}$ is the interpolation function between two points p_i and p_{i+1}, σ_x and σ_y are horizontal and vertical components of stress in a specific point. There may be inverted layers in initial modeling stage or failure stages (model solve stage), and several free faces may be presented above the analysis point. The algorithm that calculates forces acting on a block selects closest existing face above the analysis point for calculation of stresses (Fig. 6(b)). This face changes through different stages of analysis with the development of failure.

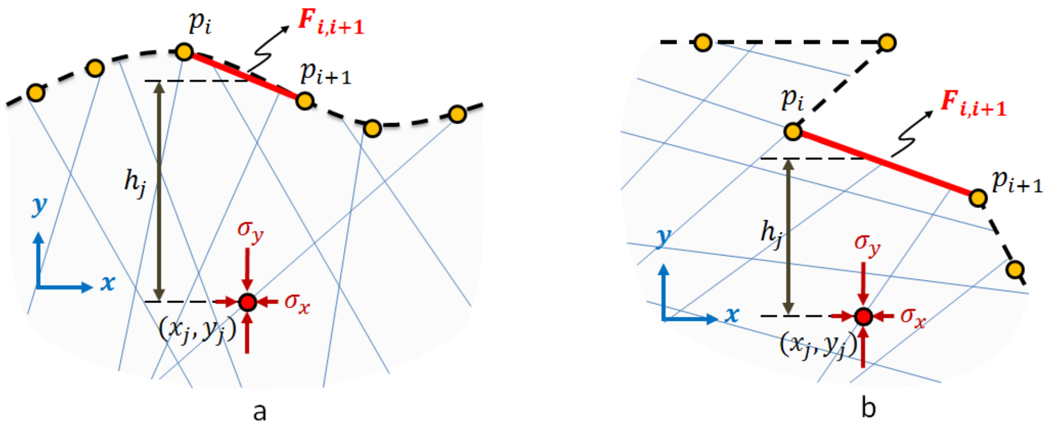


Figure 6. Schematic of stress calculation method for a point located in rock mass. a) normal free surface b) inverted free surface

In solving limit equilibrium TFS_KGM, distributed stresses on each edge should be divided to normal and shear distributed stresses and then be converted to two equivalents normal and shear forces and act on an appropriate point located on the edge.

According to Fig. 7 suppose a block with an edge with length and thickness of L and T respectively and angle of θ is present in the rock mass. Eqs. (6)-(10) are used for calculation of two equivalent normal (F_n) and shear (F_s) forces acting on this edge and also the calculation of point of application (L_p) of these two forces on the edge.

$$\bar{\sigma}_n = 1/2\left[\sigma_x(x_c, y_c) + \sigma_y(x_c, y_c)\right] + 1/2\left[\sigma_y(x_c, y_c) - \sigma_x(x_c, y_c)\right]\cos 2\theta \quad (6)$$

$$F_n = \bar{\sigma}_n \times L \times T \quad (7)$$

$$\bar{\sigma}_s = -1/2\left[\sigma_y(x_c, y_c) - \sigma_x(x_c, y_c)\right]\sin 2\theta \quad (8)$$

$$F_s = \bar{\sigma}_s \times L \times T \quad (9)$$

$$L_p = L_c + I_c/(L_c \times T \times L),\ L_c = h_c/\sin\theta,\ I_c = (T \times L^3)/12 \quad (10)$$

where $\bar{\sigma}_n$ and $\bar{\sigma}_s$ are the average normal and shear stress respectively, F_n and F_s are the equivalent normal and shear forces respectively, θ is deviation angle, I_c is the moment of area, L_p is the diagonal distance between the free face and stress point.

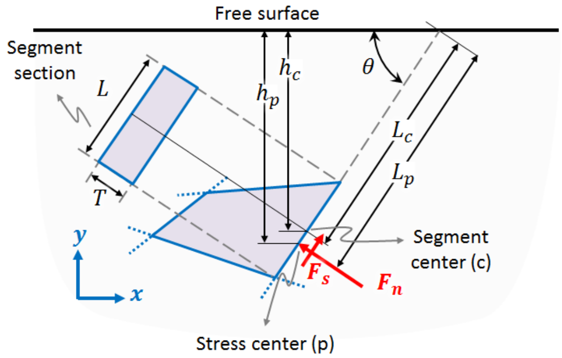


Figure 7. Demonstration of stress centroid of block edge and related parameters

As mentioned before, except for inter-block forces acting on each block edge, the bulk force of weight applies to the block as well. Here the weight force is the product of the block area and its thickness, density and gravitational acceleration are calculated and applied to the center of mass of the block. If the problem is run in quasi-static mode, a horizontal acceleration component and therefore a horizontal component of the bulk force is applied to blocks which result in the diversion of the weight force vector. Fig. 8 schematically shows the output of this procedure in quasi-static conditions on a block with horizontal acceleration coefficient of 0.3g and horizontal stress coefficient of 0.33 and a density of 2600 kilograms per cubic meter.

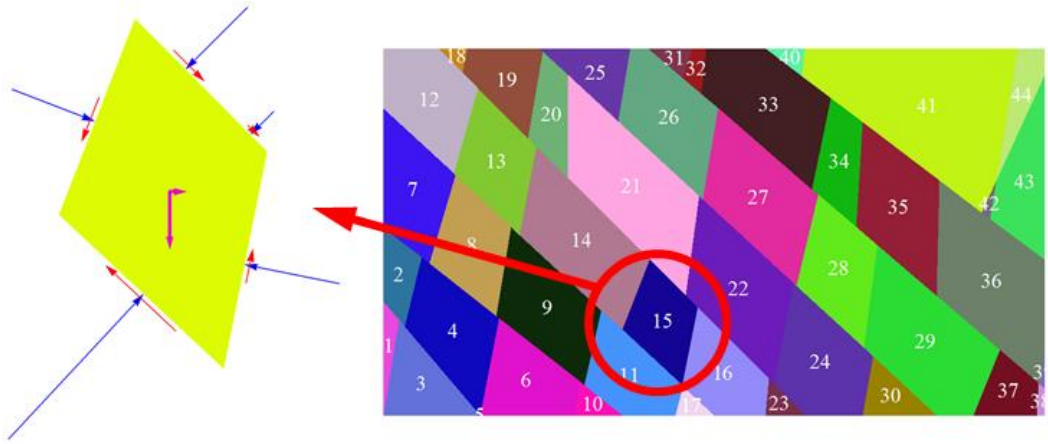


Figure 8. Demonstration of acting forces on a block located in rock mass

Blocks in a block set are in a naturally stable state in initial conditions of a problem (horizontal topography). Therefore, the resultant of all forces and moments of the body should be zero in such conditions (Eq. (11)). This means that no block can undergo inherent rotation or movement.

$$\sum_{i=1}^{n} M_i = 0, \sum_{i=1}^{n} F_i = 0 \quad (11)$$

Since blocks are rigid in TFS_KGM analysis, distribution of in situ block forces must satisfy equilibrium conditions. In other words, the resultant of all moments around all block vertices and resultant of all forces around edges are equal to zero.

The authors have investigated this subject in this study and verified it by reaching equilibrium using the proposed procedure. Based on this procedure, inter-block forces and point of the application has been verified.

2.3. Calculation of Block Mobility

TFS_KGM needs to check the possibility of development of various kinds of geometrical movements of concave and convex blocks and groups to investigate possible failure modes. A group may have various kinds of sliding, toppling and free fall instability if other neighboring blocks permit the required degree of movement and they do not collide. Determination of geometrical possibility of movement can be accomplished in different ways. Graphical and vector methods are of two most popular methods that have been used to measure sliding movement of a block[6,7]. However, the extension of these methods to rotational and free-fall movements of blocks, or considering convex and concave groups is ambiguous. In this study, block movement is investigated based on a non-vector geometrical setup to check for block collision.

In assessing the capability of the geometric mobility, first, the assessing intersection points are designed and placed in the group. Location of these points is in the center angles of vertices and center of edges with the constant distance of ε_l (Fig. 9). The sliding movement around an edge, toppling around a vertex and free fall of a group, besides defining a transfer vector component in the direction of the studied movement extends minor sliding movement ε_s, angular rotation ε_r or slight free fall ε_f, in a block or a group. The intersection of the under study block or group with neighbors is investigated in assessing points (Fig. 10). The values of parameters ε_l, ε_s, ε_r and ε_f are defined by the user.

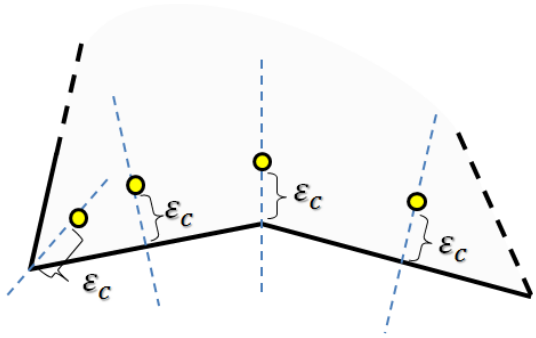


Figure 9. Schematic of locating procedure for assessing points of blocks collision

Since sliding, toppling, and free-fall movements are allowed in TFS_KGM, a group may show 8 different kinds of movements "sliding (S), Toppling (T), Free-fall (F), Sliding-Toppling (S-T), Sliding-Free fall (T-F), Sliding-Toppling-Free fall (S-T-F), and no movement (Stable)". The TFS_KGM first checks for the mobility of any block or group, if the movement is geometrically possible, a safety factor would be calculated for the possible movement in the related edge or vertex. Fig. 11 shows the output of this procedure for one of the blocks of the hypothetical block set. In sliding state (Fig. 11(b)), the negative sliding direction (red color) and in rotating state (Fig. 11 (c)) rotation in the positive direction (blue color) are geometrically moveable. Generally, mobility mode for this block is "sliding-toppling".

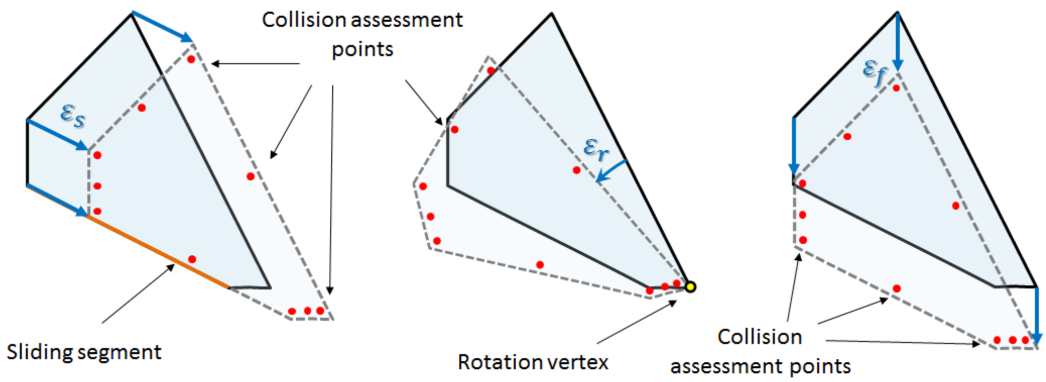


Figure 10. presentation of slight movements in order to assess block collision (εs: sliding movement, εr: rotational movement, εf: free fall movement)

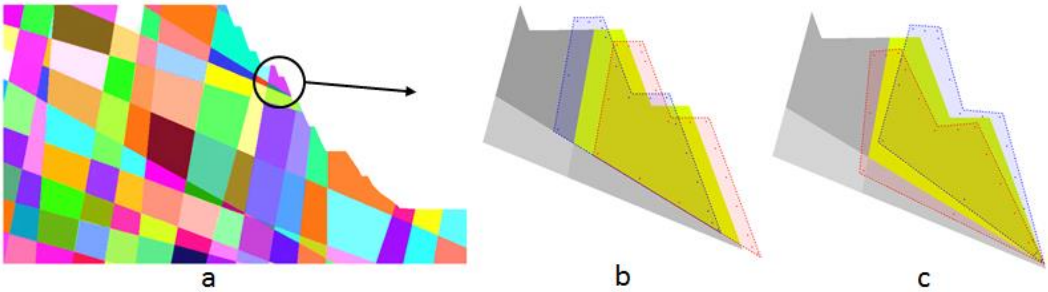


Figure 11. Allowed movements of a block in a block set (exaggerated)

2.4. Calculation of Block Safety Factor

Described equilibrium conditions result in very stable safety factors (infinite values) for the initial model. Under these conditions, any external factor that disrupts the equilibrium of forces and moments of the blocks reduces the safety factor and if the block is geometrically moveable, the block may become unstable. One of the most important external factors is the excavation of some parts of the rock mass. In the TFS_KGM, the safety factor is calculated in three general patterns of sliding, toppling, and free fall. These unstable patterns are just for limit yielding conditions and subsequent conditions are not considered.

A sliding safety factor of a block may be calculated around each edge of the blocks in both directions (positive or negative). In this case, all forces acting on a block are divided into vertical and tangential components F_{ni}, F_{si} respectively over the edge. In sliding safety factor calculations, these two forces are accounted for driving and resistant forces to sliding movement around the edge. F_{si} is the driving shear force that tends to move the block in the selected direction on the edge.The resisting forces are the shear strength (τ) and the tensile strength (F_t) in the opposite direction of the movement on all joints that would open in case of movement of the block (Fig. 12 a).

In the TFS_KGM, Mohr-Coulomb and Barton-Bandis shear strength criteria may be used to calculate the shear strength of the sliding surface. In this case, sliding safety factor may be calculated based on the following equations by dividing the resisting to driving forces [21]:

$$SF_{S-Mohr} = (\sum F_{ti} + C.T.L + \bar{F}_n\,.\,tan\,\emptyset)/\bar{F}_s \qquad (12)$$

$$SF_{s-Barton} = \frac{1}{\bar{F}_s}\sum F_{ti} + \bar{F}_n.\,tan\left[\emptyset + JRC.\,Log\left(\frac{JCS}{\bar{F}_n/(T.L)}\right)\right] \qquad (13)$$

In the above equations F_{ti} is tensile force projected in the direction of the sliding edge in the ith detached joint, C cohesion, $\emptyset$ friction angle, T model thickness, L joint length, $\bar{F}_n$ resultant of normal forces and $\bar{F}_s$ resultant of shear forces acting on the joint, *JRC* joint roughness coefficient, *JCS* joint compressive strength, $SF_{s\text{-}Mohr}$block safety factor using Mohr-Coulomb criterion, and $SF_{s\text{-}Barton}$ block safety factor from Barton-Bandis criterion. Although only two sliding criteria are used, it is possible to easily add other movement independent sliding criteria to TFS_KGM.

In order to calculate the safety factor of a block for rotation about an arbitrary corner, acting moments of all boundary and bulk forces around that corner are calculated. In this case moments depending on mobility orientation, are divided to driving moment (M_d) and resisting moments (M_r) around the corner. Apart from the external and bulk forces acting on a block, the opening of joints due to rotation of the block results in tensile forces. These tensile forces in return create resisting moments which tend to stabilize the block. Fig. 12 b demonstrates the schematic of these forces. In this case, the rotation safety factor around the corner of the block is estimated by Eq. (14).

$$SF_T = (\sum M_{ti} + \bar{M}_r)/\bar{M}_d \qquad (14)$$

Where $M_{ti}$is resisting moment due to tensile force in the ith joint, $\bar{M}_r$ resultant of all resisting moments due to the boundary and bulk forces, $\bar{M}_d$ resultant of all driving moments due to the boundary and bulk forces and SF_tsliding safety factor around the assumed corner.

In free fall movement of a block, resultant of boundary and bulk forces $\bar{F}_d$ of the block is calculated as a driving force in the direction of resultant ground acceleration gravity. Resisting forces in this instability pattern include only tensile strength of joints that open during free fall failure (since no sliding development occurs). Components of tensile forces (F_{ti}) projected in the direction of free fall are calculated as resisting forces (Fig. 12 c). In this case, the safety factor is calculated using Eq. (15).

$$SF_F = \sum F_{ti}/\bar{F}_d \tag{15}$$

F_{ti} is the projected tensile force in the direction of free fall from the ith opened joint, $\bar{F}_d$ is the resultant boundary and bulk forces in the direction of fall, and SF_F is the safety factor of the falling block.

In the TFS_KGM, at any stage, sliding safety factors around all edges of a block or group are calculated in both positive and negative directions. Also, rotation conditions around all block corners (in both clockwise and counter clockwise) and free fall conditions in the direction of the resultant gravity force and horizontal acceleration is examined. Then, impossible movements are removed and safety factors of other kinds of movement are calculated. The movement with the least safety factor is selected as the unstable block movement and the related safety factor is selected as the block safety factor.

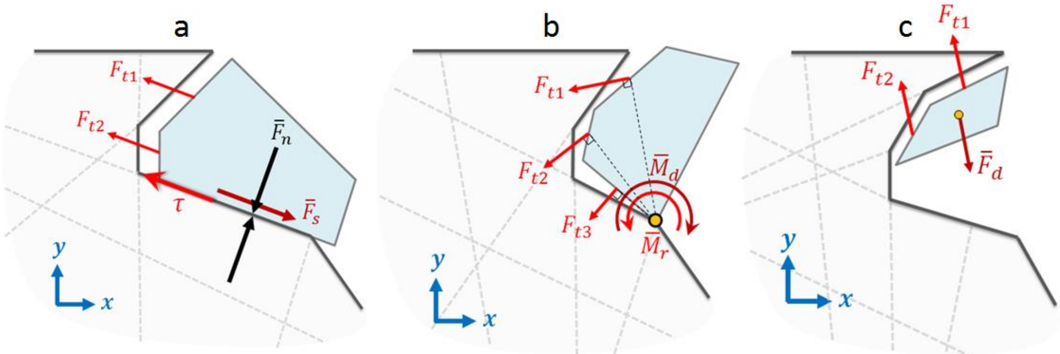


Figure 12. Schematic representation of driving and resisting forces in development of a sliding, rotating, and free fall movements

2.5. Calculation of Key Blocks

The main idea of KBM, KGM, and TFS_KGM is based on finding key blocks. A key block is active (which has a common boundary with excavation surface), finite, geometrically moveable, and movement of this block may result in movement of neighboring blocks. To identify key blocks in TFS_KGM, the active surface of the excavation is defined by the user and it is updated through subsequent stages of the analysis. Definition of the excavation surface enables the method to identify active blocks of the block set. The mobility of active blocks is examined and key blocks with their movement (one of the 8 mentioned possible movement modes) are identified. Fig. 13 demonstrates the calculation of key blocks in two hypothetical sets.

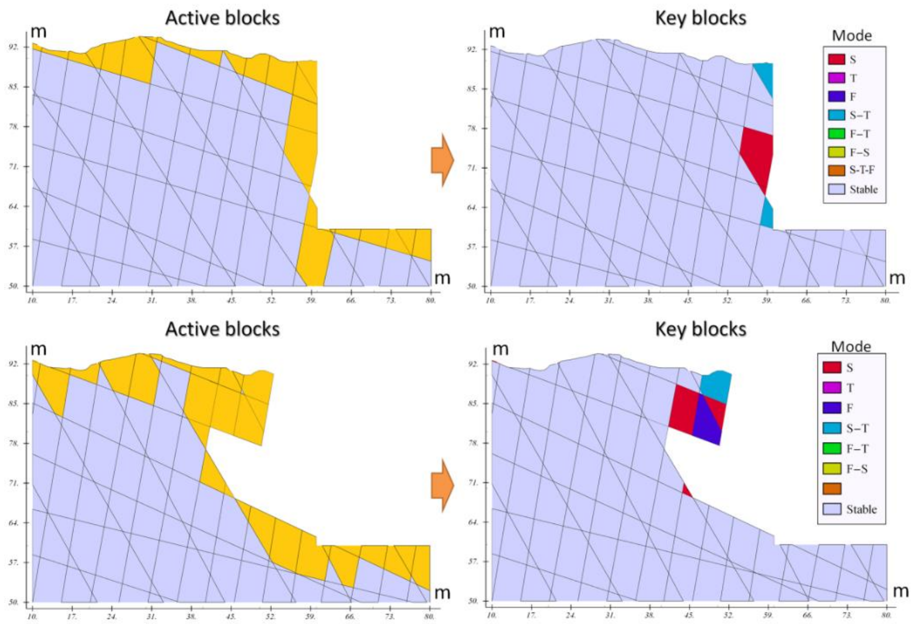


Figure 13. Key block identification process in TFS_KGM

2.6. Stepwise Strategy of Analysis in TFS_KGM

TFS_KGM uses a stepwise analysis procedure. The analysis begins with the definition of (active) excavation surface by the user. It enables the method to identify the active blocks and subsequently key blocks and their movement mode (one of the 8 mentioned possible movements) in the next step. Since the resultant normal and shear forces in excavation boundaries are equal to zero, after the loss of equilibrium, block forces are recalculated and in the meantime, safety factors of the boundary blocks are reduced. Then, the key block with the lowest sliding, rotating, and free fall safety factors is selected for the development of grouping procedure in the next step.

Grouping procedure considers neighbors of the selected key block. In the next step, groups consisting of the key block and its first-degree neighbors are formed and safety factors and mobility conditions of the groups are calculated. The group with the lowest safety factor is selected as the key group. A larger key group with a similar procedure is found in subsequent steps. The procedure continues until the lowest possible safety factor for the key group is found. The final key group is the most critically stable group. If the key group safety factor is lower than the designed safety factor defined by the user, the key group is removed (excavated) and a new excavation surface, active blocks, and key blocks are formed. The in situ stresses are redistributed and all steps are repeated. The procedure continues until the last critical group is found. Fig. 14 shows the analysis flowchart of TFS_KGM.

2.7. An Example of Solving Process of TFS_KGM

In order to demonstrate the analysis procedure of TFS_KGM, an analysis is performed on a steep trench in a jointed rock mass with geometrical conditions of toppling failure containing two sequential discontinuity sets. Geometrical and mechanical properties of the model are shown in Tables 1 and 2. Also, Fig. 15 illustrates the geometry of the problem.

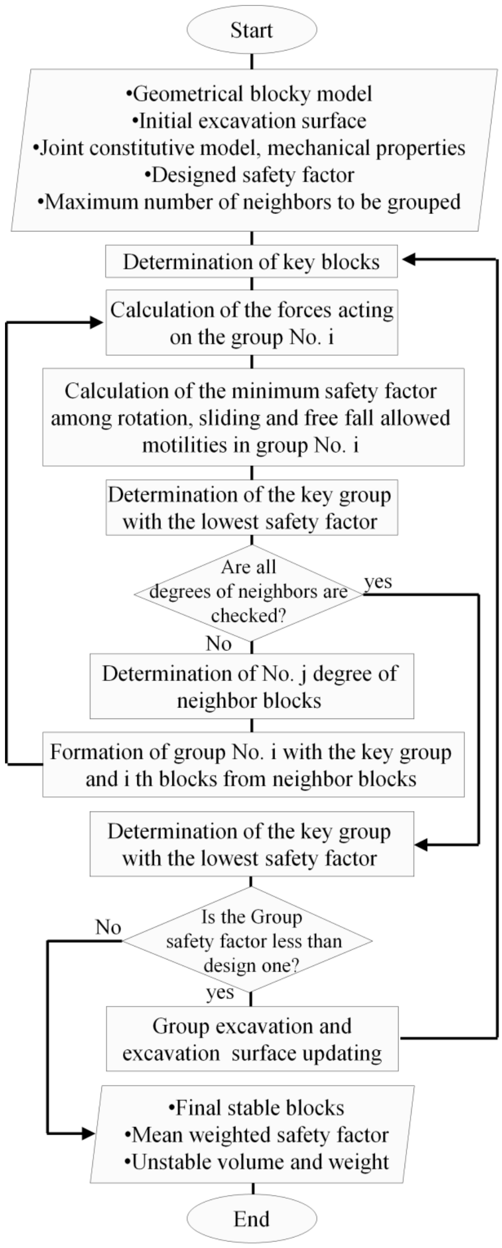


Figure 14. Analysis flowchart in TFS_KGM

Table 1. The required geometrical and mechanical properties of the vertical excavation in TFS_KGM

Mechanical parameters		Overall geometry parameters	
Criterion	Mohr-Coulomb	Wall height (m)	330
Density (Kg/m3)	2600		
Joint cohesion (Pa)	100000	Model width (m)	1
Joint friction angle (°)	35		
Joint tensile strength (Pa)	10000	Wall dip	73.14
Horizontal stress coefficient	0.33		
Gravity acceleration (m/s2)	10	No. of neighbor examination	7
Horizontal acceleration (m/s2)	0		
Design safety factor	1.5	No. of joint sets	2

Table 2. Geometrical and mechanical properties used for verification in models a, b, c and d

	Validation Model No.				Dimension
	a	b	c	d	
Density	2600	2600	2600	2600	Kg/m^3
Cohesion	20000	50000	25000	5000	N/m^2
Friction	25	35	25	33	°
Tension	0	10000	10000	10000	N/m^2
Normal Stiffness	10e9	10e9	10e9	10e9	$N/m^2/m$
Shear Stiffness	5e9	5e9	5e9	5e9	$N/m^2/m$
Stress ratio	0.5	0.33	0.33	0.33	-
Grouping attempt	10	15	10	10	-
Slope Height	63.88	32.67	63.88	64.19	m
Design Safety Factor	1.1	1.1	1.1	1.1	-

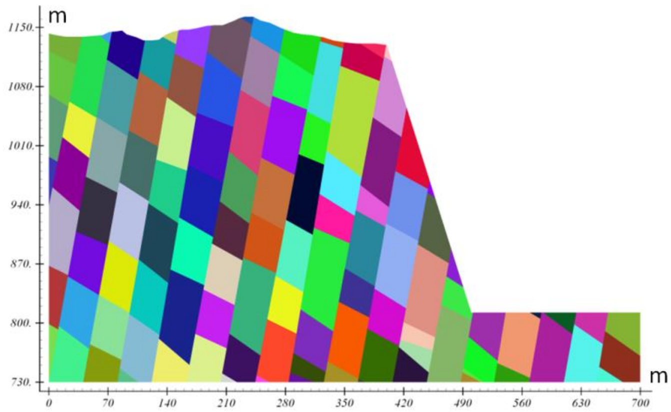


Figure 15. Initial geometry of the model after excavation and before performing TFS_KGM analysis

Figs. 16 and 17 show the results of the analysis. In this analysis, 5 stages of instability development have occurred. In the first stage, Blocks no. 1, 2, and 3 are identified as the key blocks with sliding potential. The lowest possible safety factor of these key blocks with 7 degrees of neighbors was observed in key block no. 1. Therefore, the first stage of failure was, sliding failure of the key group associated with key block no. 1 with a safety factor of 1.46 which results in instability of 9425 KN of the blocks. In the second stage, the remaining two key blocks were examined for possible instability. After examining 7 degrees of neighbors, a group consisting of blocks no. 2, 4, 5, and 6 exhibited the lowest safety factor (0.492) for a rotation movement. Therefore, a toppling failure with 163997 KN of the unstable blocks was recorded.

In the third stage, three key blocks of no. 3, 7, and 8 with a geometrical potential of sliding failure mode were detected. A group including blocks no. 3, 7, 8, 9, 10, 11, and 12 was detected with the lowest safety factor of 0.452 for sliding failure mode. This group had a weight of 326757 KN and could slide downward. The only key block in the fourth stage was block no. 13 with sliding-toppling (S-T mode) failure geometrical potential. After the examination of the possible groups of the key block and its neighbors, the key block alone was detected as the most unstable group for toppling failure. So block no. 13 with 66905 KN weight with 0.61 safety factor was removed at this stage. In the last stage, block no. 14 with sliding-toppling (S-T mode) potential was detected as the only key block. The combination of blocks no. 14, 15, 16, and 17 formed the most unstable group in a toppling mode with a safety factor of 1.11 which causes the removal of 122935 KN of the remaining blocks. Fig. 16 shows the key blocks in each stage with different colors. It also shows the final unstable state in several stages of the analysis. In addition, the distribution of gravitational stresses in each stage of failure is shown in Fig. 17.

Finally, the weighted safety factor of unstable groups and blocks for different stages of the analysis is calculated based on their related volume using Eq. (16).

$$MWSF = \sum_{i=1}^{n}(SF_i * V_i) \Big/ \sum_{i=1}^{n} V_i \qquad (16)$$

where $MWSF$ is the mean weight of safety factor, SF_i is the minimum safety factor of the ith stage for sliding, toppling, or free fall failure of a group or a block, and V_i is the volume of the unstable group in the ith stage. The final results of the analysis are shown in Fig. 18.

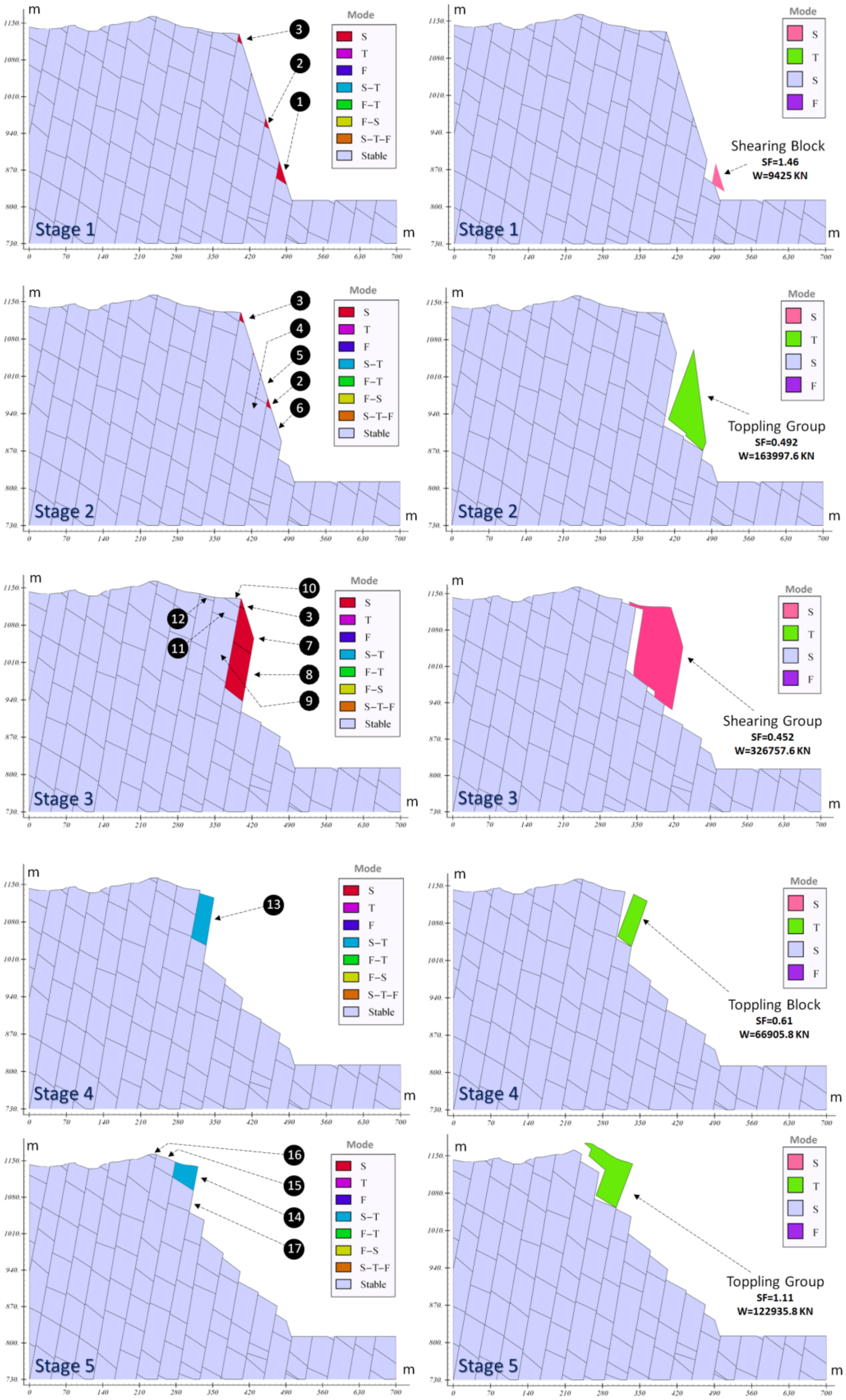


Figure 16. Key blocks position and final instability pattern in different stages of the analysis using TFS_KGM

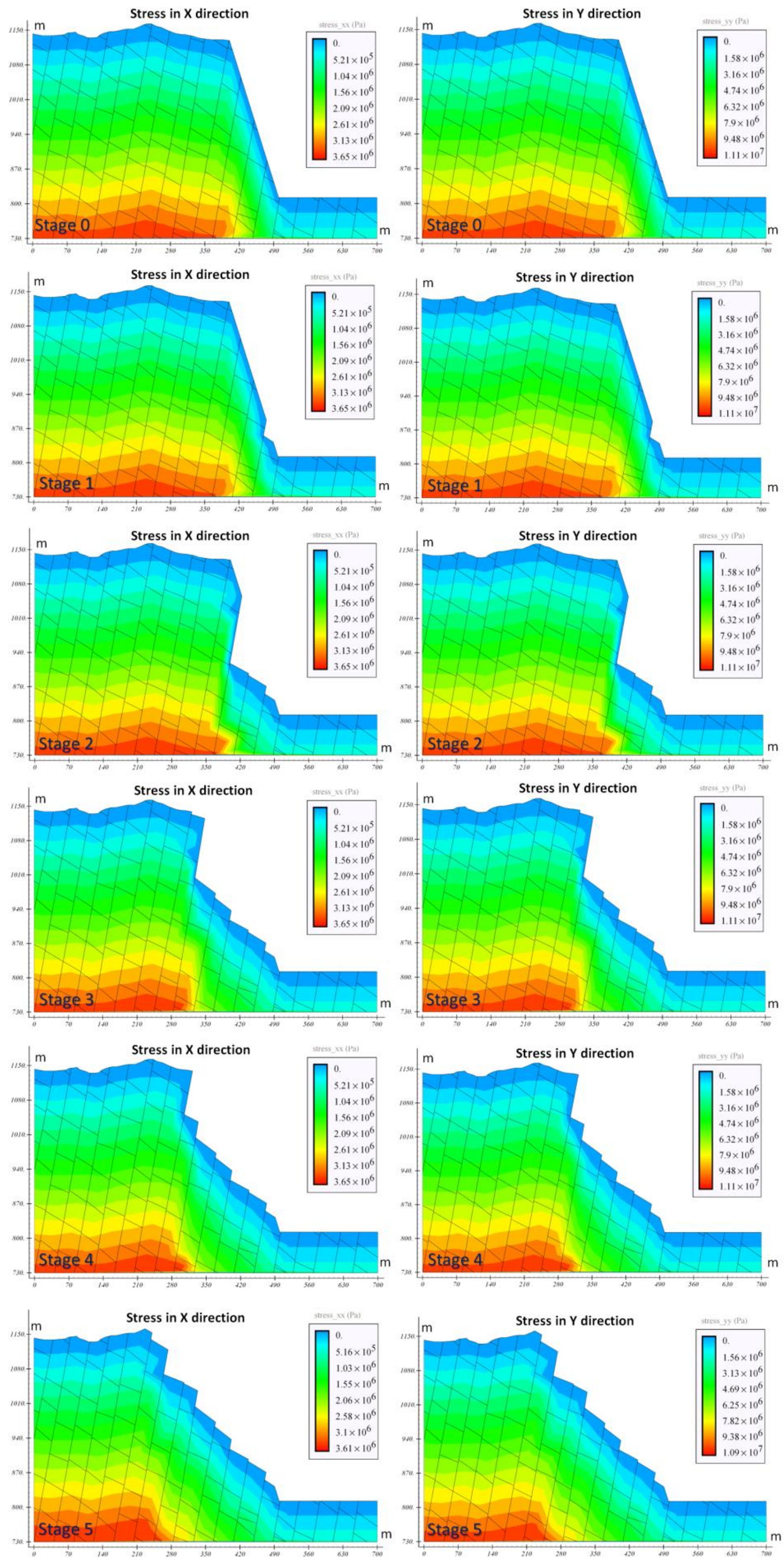
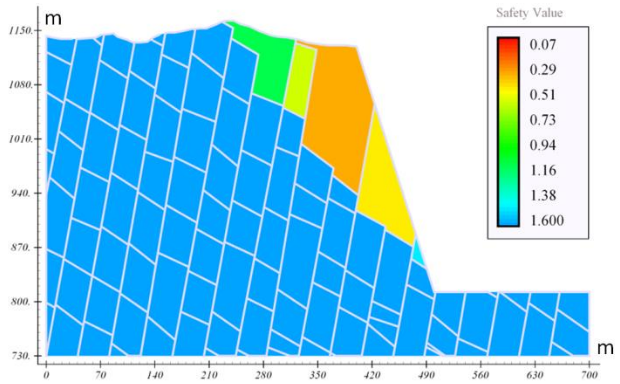


Figure 17. Contours of horizontal and vertical stresses in different stages of TFS_KGM analysis



Detail of results

Parameter	Value
Mean Weighted Safety Factor	0.6037
Total instable Weight (KN)	690021.8

Figure 18. The final unstable state and safety factors of the studied model

3. COMPARISON OF THE RESULTS WITH PREVIOUS METHODS

The result of TFS_KGM (which is able to identify toppling and free fall failures besides sliding failure) is compared with key block method (KBM) and key group method (KGM). In the first simple model, 7 regular rectangular blocks have been analyzed in a sequential column on a 45° inclined surfaces. In this model, the cohesion is zero, friction angle is 35 degrees, tensile strength of the surfaces is 10000 Pa, and material density is 2500 Kg/m^3. The problem is solved for design safety factor of 5. The results of this analysis are shown in Fig. 19.

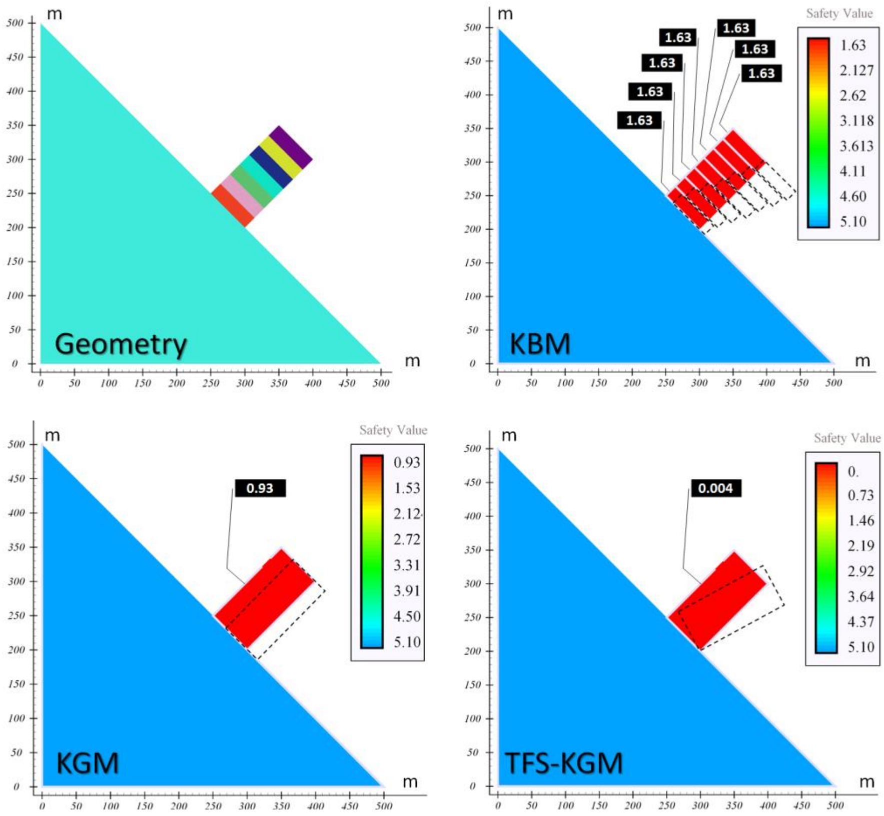
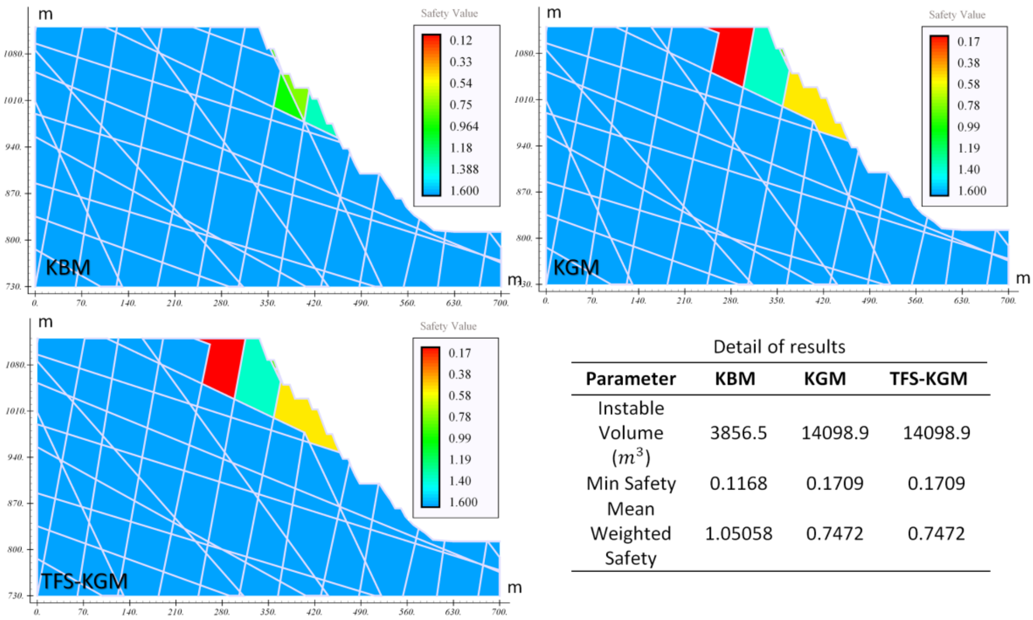


Figure 19. Comparison of the results of safety factor and instability pattern in key block method (KBM), key group method (KGM), and toppling-free fall-sliding key group method (TFS_KGM) for a simple inclined column containing 7 blocks

Significant differences were observed between the results of analyses. The results provided by the KBM, considering that grouping was not possible, blocks from the top of the column, were key blocks and exhibited sliding instability. The KBM analysis leads to an average safety factor of 1.63. While solving the same problem with the KGM, provides the possibility of combining the blocks and grouping of them. However, because the rotational movement is not considered in this model, and the lack of possibility of toppling failure, in this case, the safety factor is calculated based on the sliding failure only and it is equal to 0.93. With the implementation of this model in the TFS-KGM, it was possible to examine the formed group for rotational mode and accordingly a safety factor equal to 0.004 was obtained. In the second model, a slope with three sets of non-sequential joints was considered in a way that planar failure was potentially possible. In this model cohesion was 20000 Pa, friction angle was 25°, the tensile strength was 10000Pa, and a density of 2600 Kg/m³ was considered. Also, a design safety factor of 1.5 was used in the model. Fig. 20 compares the results of the analysis of the model with three approaches of KBM, KGM, and TFS-KGM.



Detail of results

Parameter	KBM	KGM	TFS-KGM
Instable Volume (m^3)	3856.5	14098.9	14098.9
Min Safety	0.1168	0.1709	0.1709
Mean Weighted Safety	1.05058	0.7472	0.7472

Figure 20. Comparison of the safety factor and unstable volume in KBM, KGM and TFS_KGM for a slope with planar failure potential.

The comparison of the results of the KBM and the TFS-KGM for the second model shows significant differences. The safety factors have almost 40% difference and the unstable volumes have around 72% difference. However the results are quiet similar in KGM and TFS_KGM. This is due to the lack of rotational or free-fall instability growth in TFS_KGM. In other word, similar to KGM, sliding is known as the most critical pattern of instability in different stages of TFS_KGM analysis.

In a third model, two sets of sequential joints with toppling failure potential were considered. In this model, cohesion was 10000 Pa, friction angle was 35°, the tensile strength was 10000Pa, and a density of 2600 Kg/m³ was considered. Also, a design safety factor of 1.5 was used in the model. Fig. 21 compares the results of the analysis of the model with three approaches of KBM, KGM, TFS-KGM.

Comparison of the results in the third model showed significant differences among all methods. The safety factors and unstable volumes in the KBM and the TFS-KGM have 106% and 93% differences, respectively. The safety factors in the KGM and the TFS-KGM have 21% difference, while giving the same unstable volumes. Although the KGM and the TFS-KGM have provided identical unstable volumes in this model, the safety factors are quite different. These results show the low accuracy of KBM and to a lesser extent KGM. For toppling failure, application of TFS-KGM shows higher accuracy and sensitivity to the geometry of the problem.

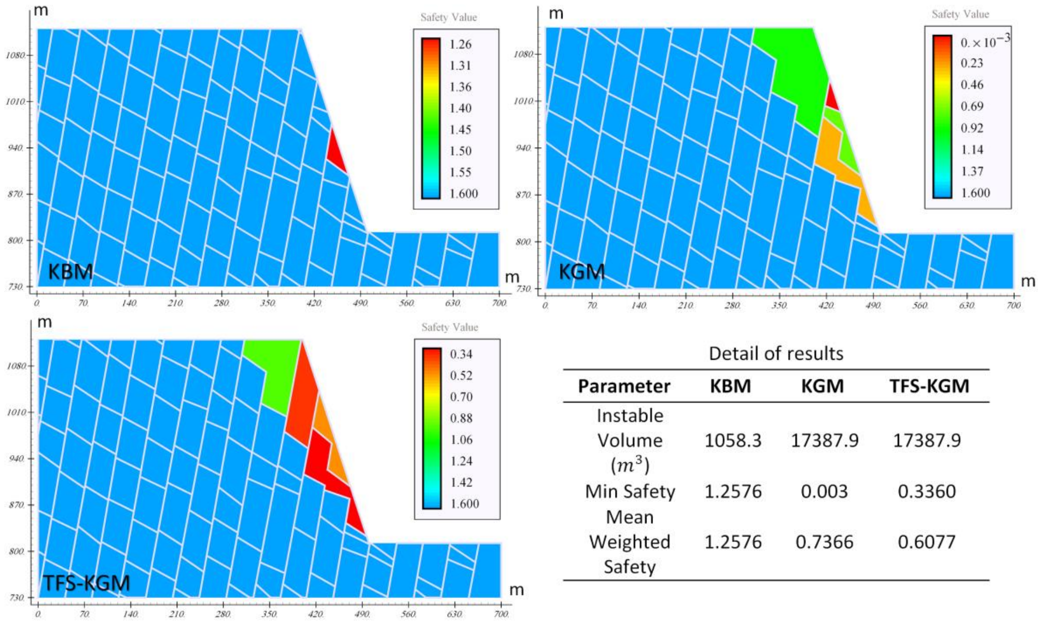


Figure 21. Comparison of the safety factor and unstable volume in KBM, KGM, TFS_KGM for a slope with toppling failure potential.

4. VERIFICATION OF THE PROPOSED MODEL

The discrete element method (DEM) programmed in UDEC (version 6) presented by ITASCA have been used to verify the proposed TFS_KGM. Four jointed slopes, namely *a*, *b*, *c* and *d* with different mechanical properties are analyzed using UDEC and the TFS_KGM. In order to synchronize the models, rigid blocks were used in the analysis of UDEC. This assumption was used because blocks in TFS_KGM are assumed to be rigid.Failure limit with 10 cm displacement is selected in DEM and blocks with larger displacements which have significant velocity are removed from the analysis. The analysis in the DEM continues to the extent where there are no more unstable blocks. Geometrical properties of the joint set of models a, b, c and d used for validation are shown in Table 2. Also, Table 3 demonstrates mechanical parameters and some analytical parameters in the models.

The unstable volume obtained by both methods was compared. Also, the minimum and the weighted mean safety factor of unstable blocks were calculated for all models. The results of the verification are shown in Figs. 22-25.

Table 3. Joint sets' properties used for verification in models a, b, c and d.

Jset No.	Dip (°)								Spacing (m)							
	Mean Angle				Std.				Mean				Std.			
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d
1	120	120	120	150	10	3	10	3	10	10	13	13	0.01	0.01	0.01	0.01
2	80	80	80	80	4	1	4	5	10	5	13	13	0.01	0.01	0.01	0.01
3	160	160	160	120	7	3	3	5	10	8	13	13	0.01	0.01	0.01	0.01

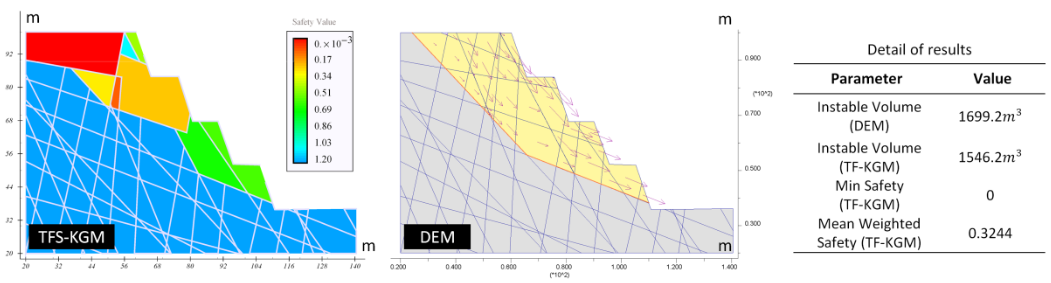


Figure 22. Comparison of instability state in TFS-KGM and DEM for model a

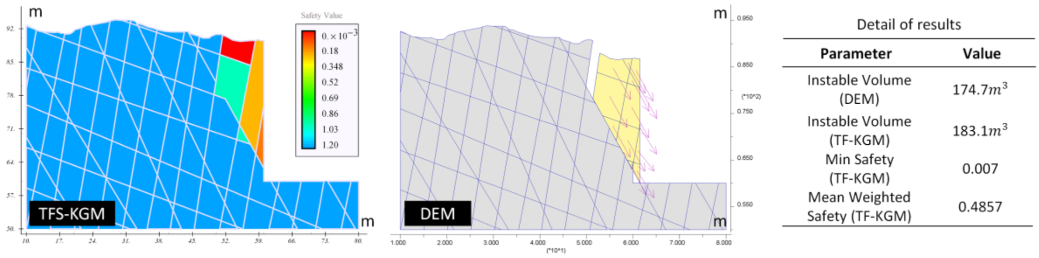


Figure 23. Comparison of instability state in TFS-KGM and DEM for model b

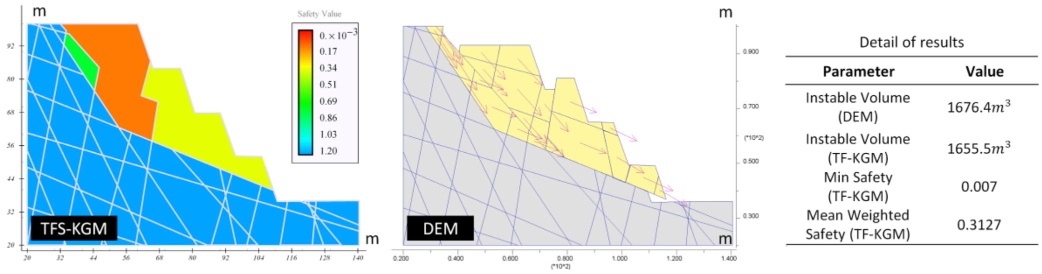


Figure 24. Comparison of instability state in TFS-KGM and DEM for model c

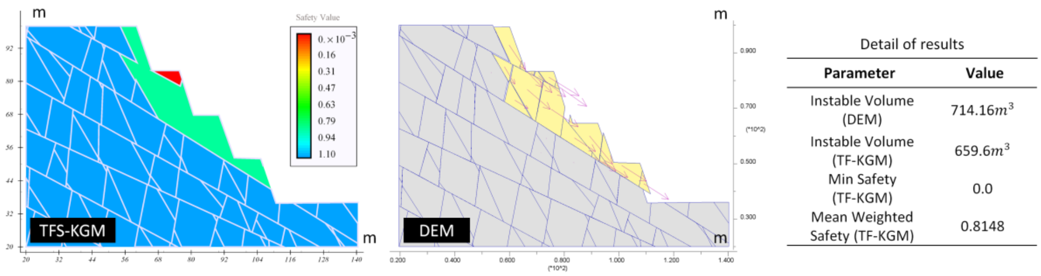


Figure 25. Comparison of instability state in TFS-KGM and DEM for model d

The conformity error values between the two methods in four models *a*, *b*, *c* and *d* are reported in Table 4. According to the table, the mean percentage of error for the results of unstable volumes in the two methods is equal to 5.65%.

Table 4. Conformity error of DEM and TFS-KGM results

Matching error of instable volume				Average
Model a	**Model b**	**Model c**	**Model d**	
9%	4.8%	1.2%	7.6%	5.65%

As many researchers have expressed in the field of rock engineering, Acceptable error threshold is equal to 10% [24] .So presented results for the DEM and the TFS-KGM match appropriately which shows the validity of the results of the TFS-KGM.

5. DISCUSSION AND CONCLUSIONS

The toppling-free fall-sliding key group method (TFS_KGM) was presented as an extension to key blocks method (KBM) proposed by Goodman et al. and key group method (KGM) proposed by Yarahmadi et al. The proposed method has the capability of developing rotation (toppling) and free fall movements in addition to sliding movement which is especially of paramount importance in toppling failure.

The comparison of the proposed method with key blocks and key group's method revealed a significant difference between this method and key block method. Also, the difference between the proposed method and key groups method is significant when toppling and free fall movements are possible. Therefore, the TFS_KGM can be recognized as a proper improvement over the previous methods.

In verification of the method, it was shown that there is a negligible difference between the instable volume of the TFS_KGM and the DEM. In other words, the proposed limit equilibrium method could present the results of the DEM (which uses a significant amount of computation) with acceptable accuracy (almost 5% error). The proposed method uses a much simpler calculation process and needs very much lesser computation time than the DEM. Therefore, applying TFS_KGM, one can achieve accurate results for stability analysis of jointed rock slopes without dealing with complicated computations and timeconsuming process of the numerical methods. In addition, one of the major advantages of this method is providing reliable safety factor. Based on this, one can estimate the stability condition in different confidence levels and with tacking in to account toppling and free fall instabilities besides sliding ones. This safety factor can be a basis for future decisions. Also, the TFS_KGM considers effects of in situ stresses in a limit equilibrium analysis.

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